Angst+Pfister

Antivibration

TECHNICAL GUIDE

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Introduction

All the various sectors of today's technically-orientated civilisation are subject to vibrations, oscillations, effects from impact and irritating noises. Better utilisation of high quality materials, light-weight structures and increasingly efficient plants have further strengthened the tendency for vibrations.
At the same time, as a result of increasing environmental influences, man has become even more sensitive to vibrations. Regulations governing the acceptable effects of vibrations are increasing all the time and our demand for greater product accuracy has given rise to the need for vibration-proof production plants.

These documents will help you reduce the vibrations produced by machines, motion, the environment and by man. Generally speaking, we must be aware of the fact that vibrations cannot be eliminated, but only reduced to a bearable and safe level.

The theoretical assumptions, explanations and examples of calculations given below, together with the descriptions of individual products, will be of help to manufacturers and users in finding an optimal solution to problems deriving from vibrations.

Solutions for reducing vibrations

Depending on their intensity and sensitivity, mechanical vibrations, solid borne noises and shocks all disturb the environment. They may be caused by various factors such as, for example, mechanical movements, aerodynamic influences, vibrations caused by traffic, building works, explosions, earthquakes or inappropriate processing methods. Even if, ideally, undesirable disturbance should be eliminated at the source, this is possible only in a very limited number of cases.

Generally speaking, a distinction is made between the two types of vibration reduction listed below.

ACTIVE ISOLATION

In the case of active insulated, oscillating or vibrating plants are insulated from the environment using appropriate isolating elements (vibration dampers) to prevent disturbance from being transmitted to the environment.

To succeed in this action the following factors must be taken into consideration: the frequency of the plant‘s disturbance (vibration), its structure, the position of its centre of gravity, the oscillation amplitude, acceptable accelerations, the plant‘s mass and the place where it is installed.

PASSIVE ISOLATION

In passive isolation delicate installations such as entire buildings, nuclear power stations, sound studios, laboratories and devices such as weighing machines and measuring apparata are shielded from vibrations, shocks and oscillations by suitable isolating elements (isolating panels, air cushions, etc.).

Symbols, units and concepts

Symbols	SI units	Other acceptable units	Concepts
A		mm^2	section, support surface
D			degree of damping, Lehr damping
E		N/mm^2	modulus of elasticity
F	N	kg	force, capacity
G		N/mm^2	modulus of tangential elasticity
H		Shore A	hardness
K		dB	damping value
KB			KB value
T	s		length of period of time
U			transmission ratio
V	m^3	dm^3	volume
W	J	Nm	work absorption
a	m/s^2	mm/s^2	acceleration, oscillatory acceleration
b		mm	width
c		N/m, N/mm	elastic constant, elastic rate, elastic characteristic
f	Hz		frequency
g	$9{,}81 m/s^2$		acceration of gravity
h			height, thickness
i			isolating performance
k			coefficient of proportionality
l			length
m	kg		mass
n	1/s		number of revs.
p	Pa		pressure, load per surface unit
p			damping percentage
q			co-efficient of shape
s	m		elastic deflection
t	s		time
t	K		temperature
v	m/s		speed, oscillatory speed
x			cartesian co-ordinates, guide, oscillatory stroke
α	W/m · K		thermal conductivity
ρ	kg/m^3		density
χ			adiabatic index
ψ			relative damping
ω	1/s		angular speed
Λ			logarithmic decrement
λ			frequency ratio, f_{err}/f_0
η			coefficient of machanical absorption, performance

Indices

ʹ	dynamic …
–	average value
^	maximum value (amplitude)
0	natural …
A	work point
C	compression
D	pressure, damping
F	foundations
M	machine
S	shearing stress
U	basic
a	axial
m	mass
r	radial
ü	transmitted …
x, y, z	axes of coordinates
err	exciter
sub	sub-tangent

The fundamentals of vibration isolation

The principle of vibration isolation is the same for both active and passive isolation. Generally speaking a mass can oscillate in any direction or in any manner.

1

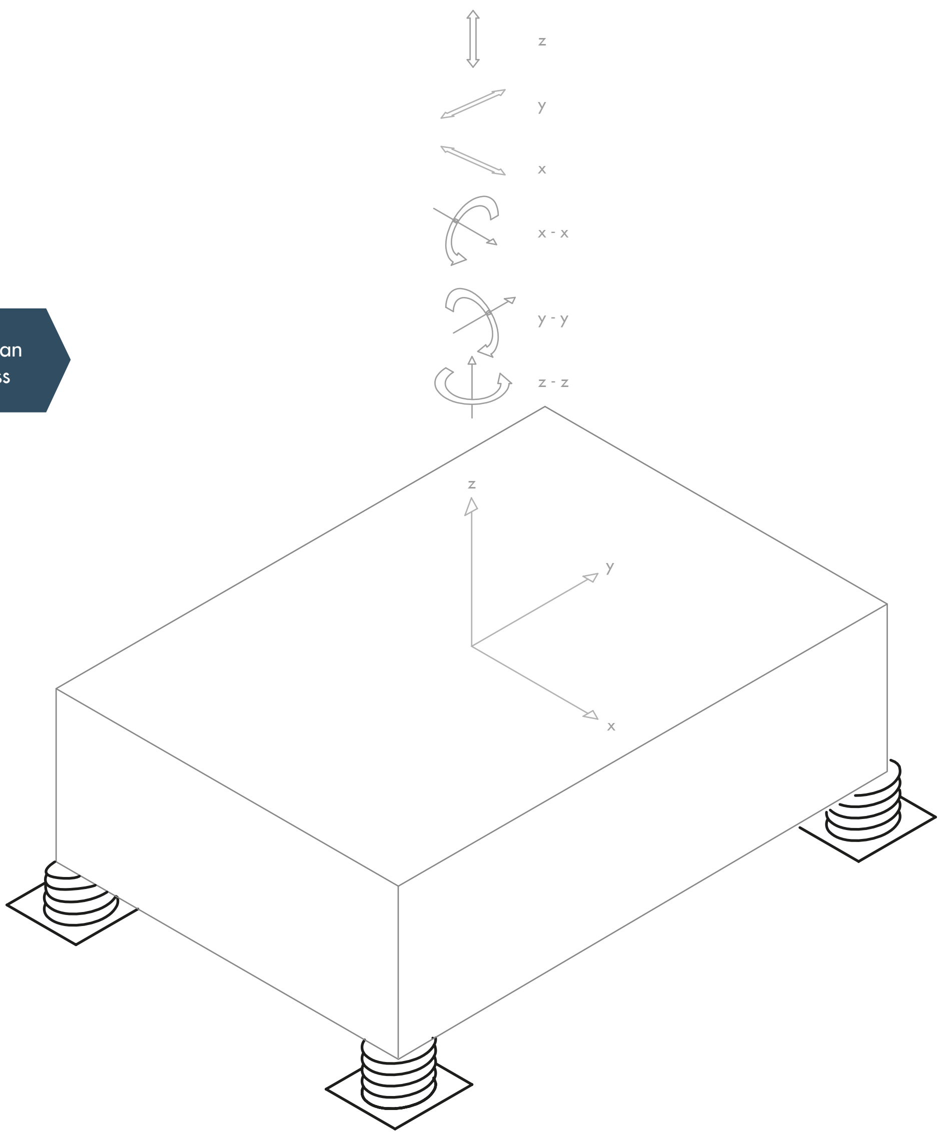
NATURAL FREQUENCY

An oscillator with a single mass supported by elastic elements has six degrees of freedom (directions of oscillation). The six natural frequencies can be calculated if the machine's mass, its dimensions, general centre of gravity and dynamic rigidity are known.

For most systems, only one or two natural frequencies are relevant as the excitation is limited to these direction. This is however heavily dependant on the type of system.

A body of this type, supported by isolating elements, if excited by a shock, will produce a sinusoidal oscillation, with a specific natural frequency.

The degree of freedom of an oscillator with single mass



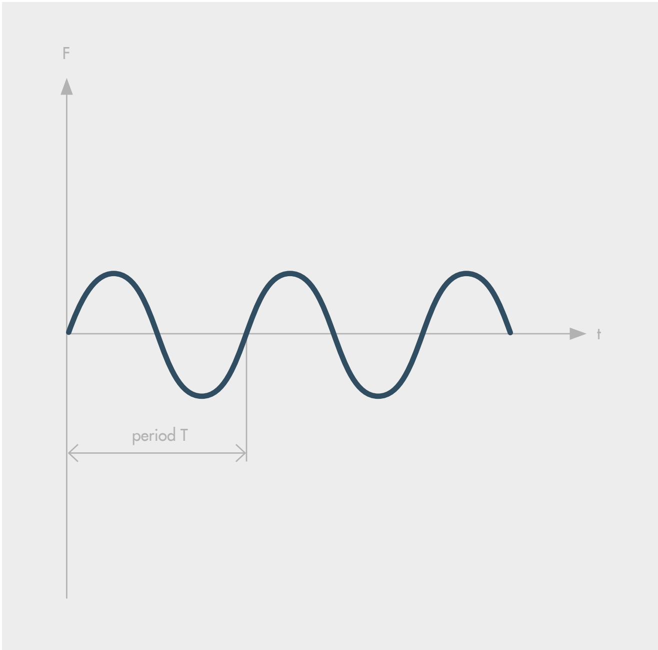
EFFECTS OF DISTURBANCE

Generally speaking the dynamic effects of a force can be divided into four different types.

- Periodic
- Harmonic
- Aleatory
- Pulse

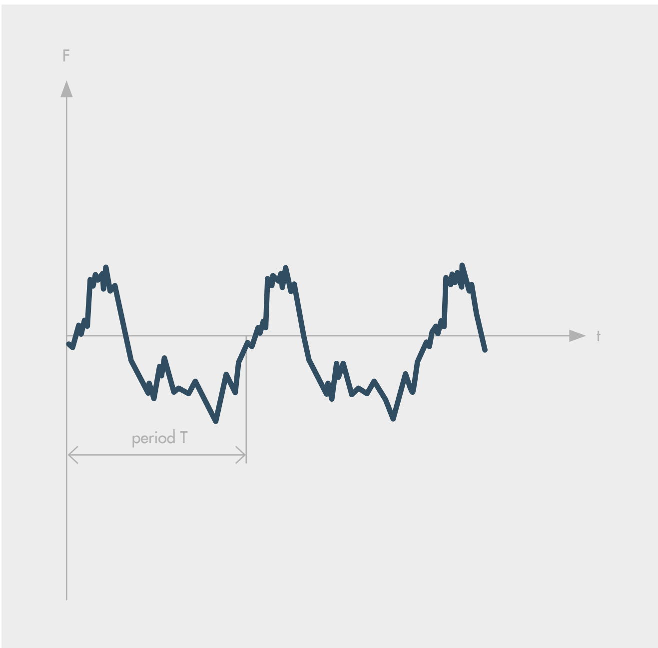
THE HARMONIC EFFECT

The harmonic effect has a sinusoidal trend, e.g. machines with rotating components can generate oscillations similar to a sinusoid. In practice, an oscillation which is only sinusoidal is rarely encountered.



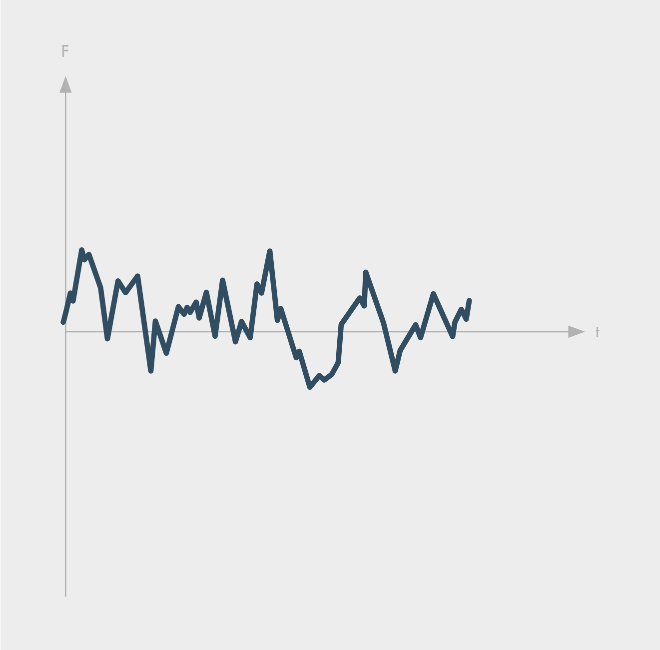
THE PERIODIC EFFECT

The Periodic effect is the most common araised effect. After carrying out a Fourier analysis it is possible to divide the periodic oscillations into individual sinusoidal oscillations in order to simplify it.



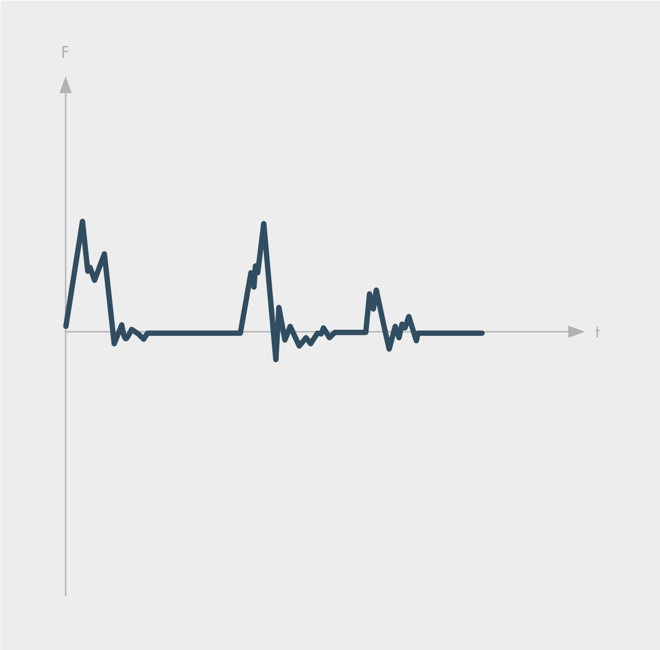
THE ALEATORY EFFECT OF FORCE

The aleatory effects of force have a trend as required which do not present any identifiable periodicity. Aleatory vibrations derive for example from the wind, earthquakes, or partially from effects caused by traffic.



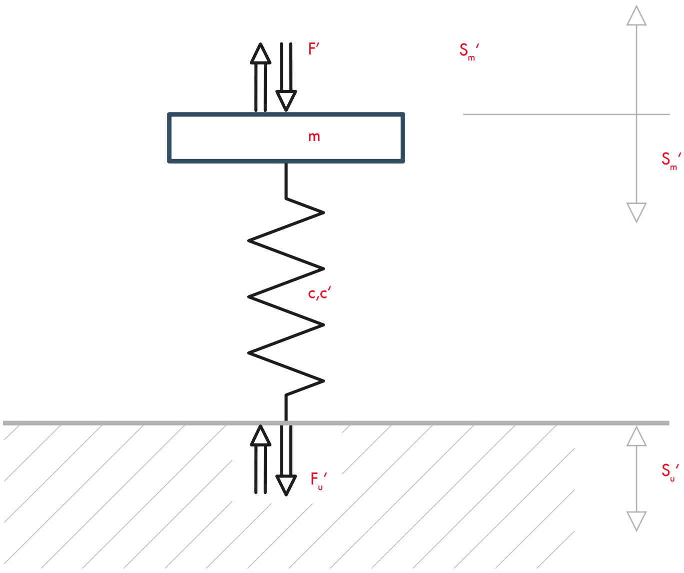
THE PULSE EFFECT OF FORCE

Pulse disturbance lasts for a much shorter time. This type of disturbance is generated following explosions, accidents or shock effects from for example forges or embossing presses.



CALCULATION MODEL

Many vibration problems can be compared approximately to a simple physical model, the so-called unidimensional mass-spring oscillator.



F'	dynamic force – external agent [N]
F_U'	dynamic force on the base [N]
m	oscillating mass [kg]
S_m	static displacement of mass [m]
S_m'	dynamic displacement of mass [m]
x_U'	dynamic displacement of the base [m]
c, c'	static or dynamic elastic constant [N/m]

NATURAL FREQUENCY

Every mass-spring system gives rise to oscillatory movements when excited. A differentiation is made between pulse excitation and continuous excitation. If a mass-spring system is excited by a single pulse it will oscillate at a natural frequency until the momentum energy adopted is transformed into heat following damping. The natural frequency f_0 because of an undamped single mass oscillator is determined by the dynamic spring constant c' and by the size of the mass. It is calculated on the basis of the following formula:

$$f_0 = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{c'}{m}} \quad [\mathrm{Hz}]$$

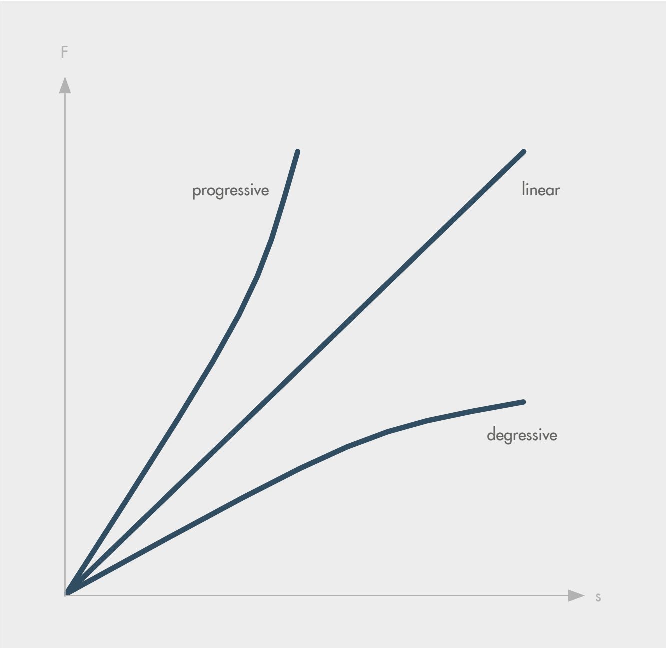
c'	dynamic spring constant [N/m]
m	mass [kg]

A direct ratio is created between the natural frequency and the elastic deflection s subsequent to mass m and the elastic constant c. Considering this fact, natural frequency f_0 can be calculated approximately on the basic of the following formula:

$$f_0 \approx 60 \sqrt{\frac{250}{s}} \quad [\mathrm{min}^{-1}]$$

$$f_0 \approx \sqrt{\frac{250}{s}} \quad [\mathrm{Hz}]$$

s	elastic deflection [mm]



ELASTIC CHARACTERISTICS

In the case of linear curves, the elastic constant c is calculated on the basis of the following formula:

$$c = \frac{F}{s} \quad [N/m]$$

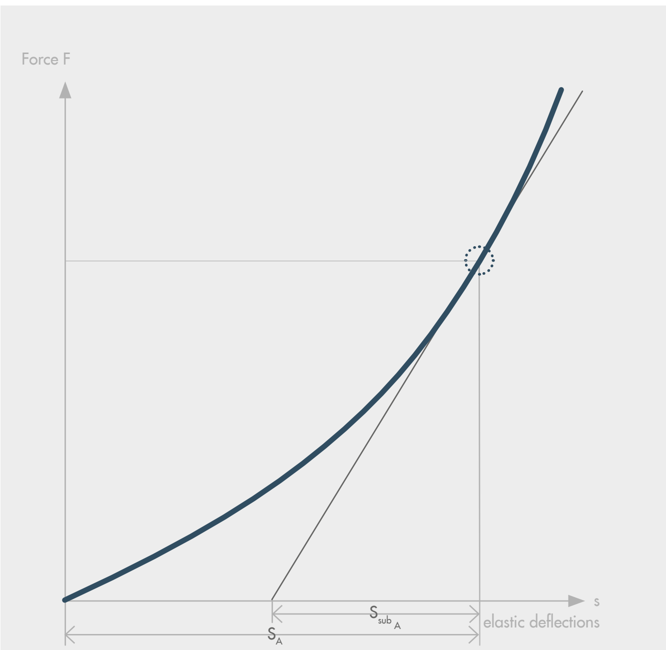
F force [N]
s elastic deflection [m]

Depending on their geometric shape and stress, vibration dampers can have increasing or decreasing curves. In these cases the elastic curve is represented by a tangent on the work point A and the elastic constant c is calculated on the basis of the following formula:

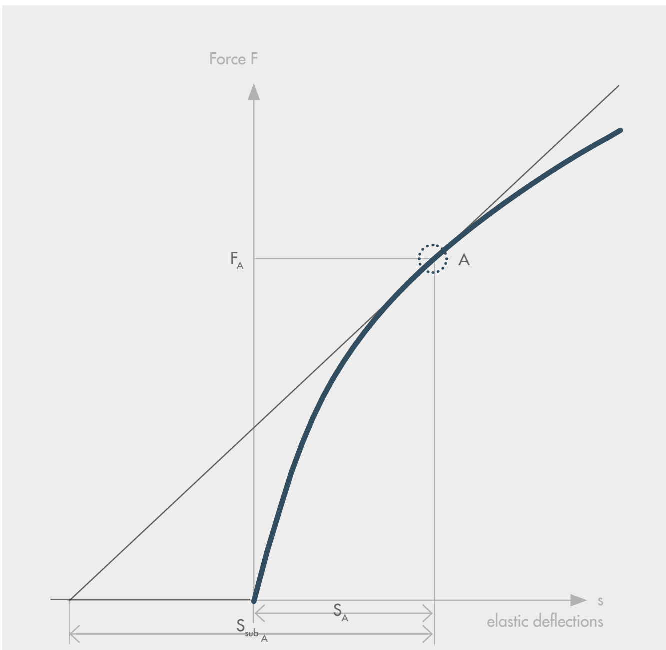
$$c = \frac{dF}{ds} = \frac{F_A}{s_{subA}} \quad [N/m]$$

F_A force at the work point [N]
s_{subA} characteristic elastic deflection at the work point A [m]

INCREASING ELASTIC CURVE



DECREASING ELASTIC CURVE

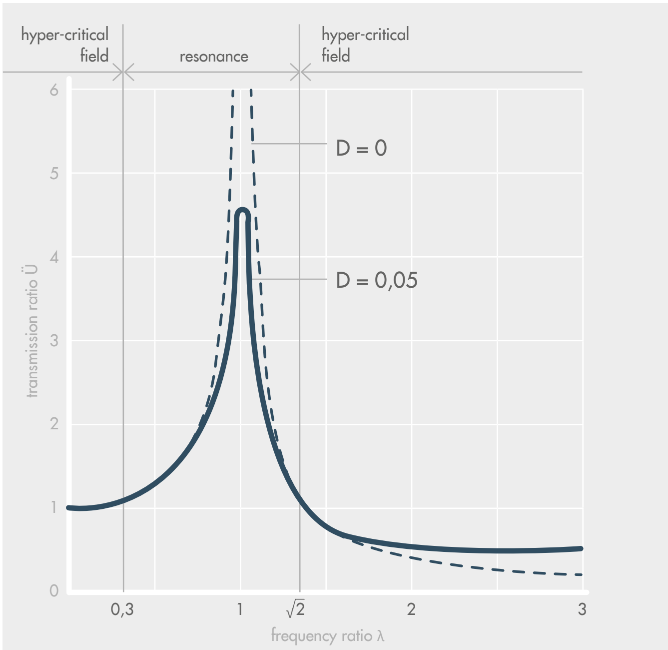


RESONANCE

A resonance of the system can occur if the excitation frequency is close to the natural frequency of the system. In this case the vibration amplitudes of the machine will be much higher than the excitations which can cause damage.

RESONANCE HAS TO BE STRICTLY AVOIDED DURING PLANNING AND DESIGNING OF THE SYSTEM !

FIELDS OF OSCILLATION

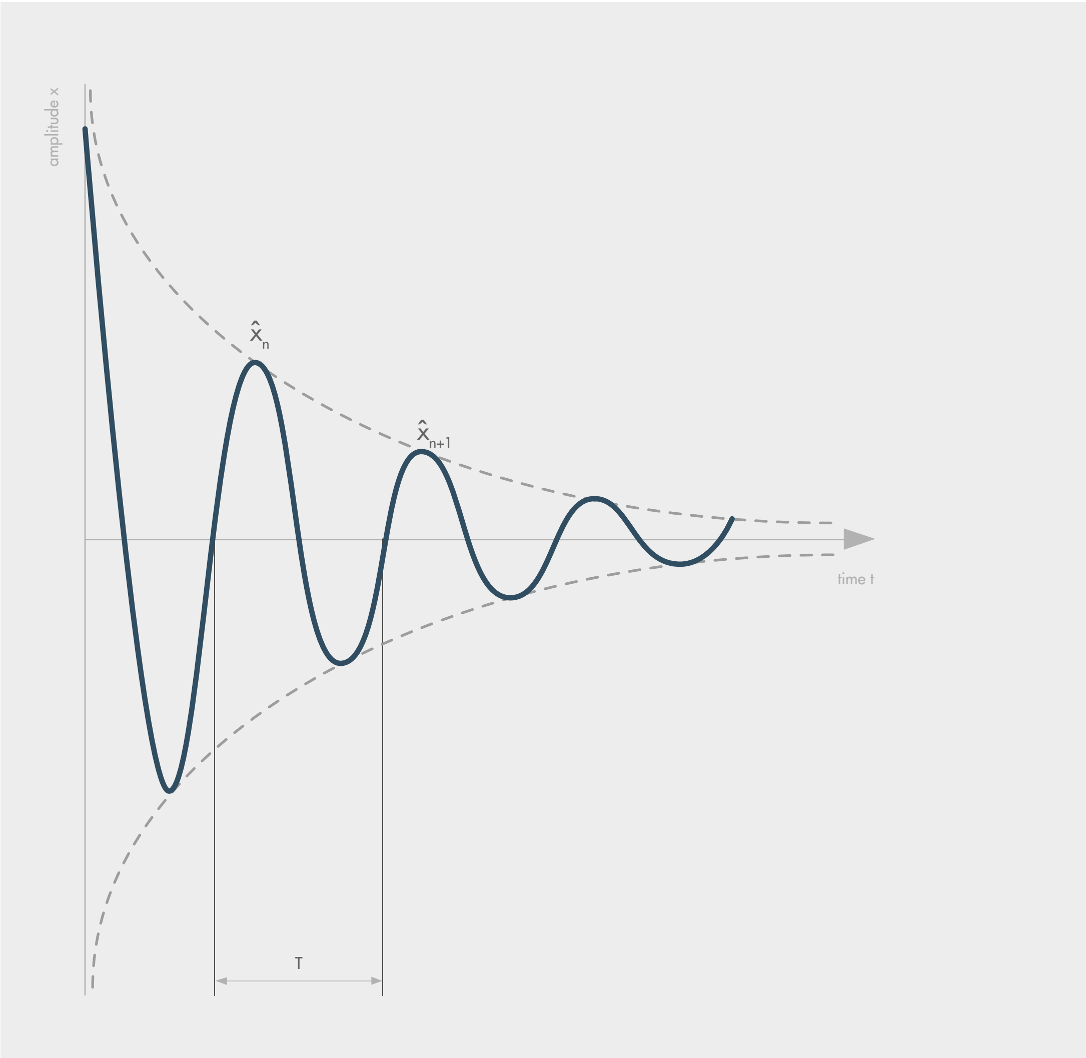


The frequency ratio λ is the ratio between the frequency of the exciter f_{err} and the natural frequency f_0. If the natural frequency f_0 is equal to the exciter frequency f_{err} (frequency ratio λ = 1), resonance will occur. In the case of a system which oscillates without damping, in the presence of resonance oscillation amplitudes would be enormous.

DAMPING

If an elastic system is transformed into oscillation following a short effect of force, these oscillations will dampen with time. The speed of damping of oscillation amplitudes depends on the damping of the support elements.

SINUSOIDAL OSCILLATION WITH DAMPING



The relationship between damping D and the ratio of two consecutive amplitudes $\hat{x}$.

$$\frac{\hat{x}_{n+1}}{\hat{x}_n} = e^{-2D\pi} = e^{-\eta\pi}$$

D degree of damping
η coefficient of mechanical loss

Steel springs have a very low coefficient of η = from 0.004 to 0.016.
Depending on the hardness and type, the loss coefficient of elastomer materials is η = from 0.1 to 0.4.

There are three different types of damping

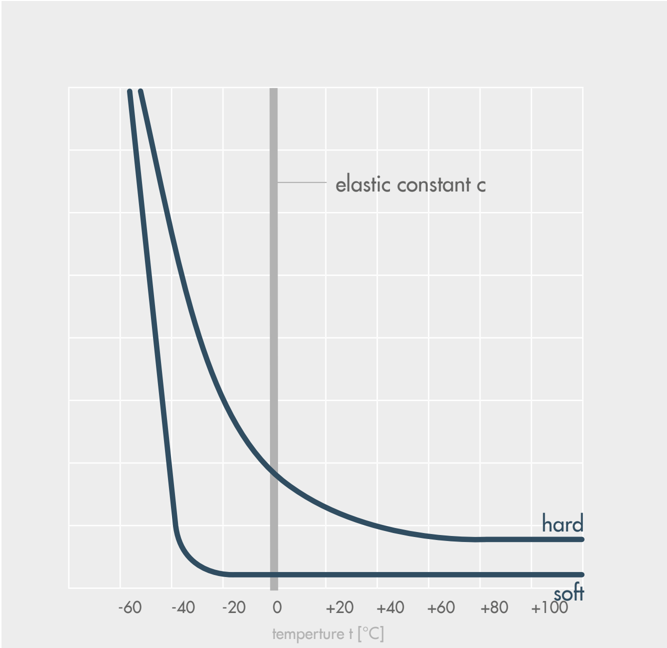
- Coulomb damping
- Hysteresis damping
- Viscosity damping

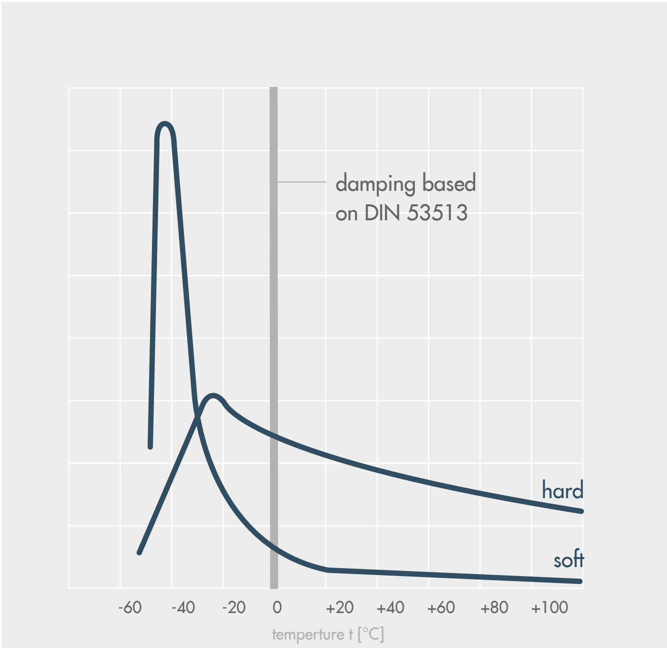
Coulomb damping is constant and does not depend on speed. Friction dampers, such as metal wool and metal cables provide Coulomb damping. Coulomb damping requires a certain tensile strength and provides suitable isolation against impact and shock.

Hysteresis isolation is based on the molecular structure of a damping material. All elastomer materials have hysteresis damping properties. Elastic damping depends on composition, hardness and temperature.

Viscosity damping is almost proportional to speed and does not depend on temperature. Silicone-based viscosity dampers have the ideal characteristics for providing isolation against oscillations and are usually used together with steel springs.

THE DAMPING AND ELASTIC RIGIDITY OF AN NR ELASTOMER REFERENCE TO TEMPERATURE





RELATIONSHIP BETWEEN THE VARIOUS CHARACTERISTIC DAMPING VALUES

Characteristic damping value	Symbol	Characteristic damping value				
		Λ	p	Ψ	η	D
logarithmic decrement	Λ =	Λ	$\frac{p}{50}$	$\frac{\Psi}{2}$	π · η	2 · π · D
percentage of damping based on DIN 53513	p =	Λ · 50	p	25 · Ψ	π · η · 50	π · D · 100
relative damping	Ψ =	2 · Λ	$\frac{p}{25}$	Ψ	2 · η · D	4 · η · D
coefficient of mechanical loss	η =	$\frac{\Lambda}{\eta}$	$\frac{p}{50 \cdot \eta}$	$\frac{\Psi}{2 \cdot \eta}$	η	2 · D
degree of damping	D =	$\frac{\Lambda}{2 \cdot \eta}$	$\frac{p}{100 \cdot \eta}$	$\frac{\Psi}{4 \cdot \eta}$	$\frac{\eta}{2}$	D

VIBRATION ISOLATION

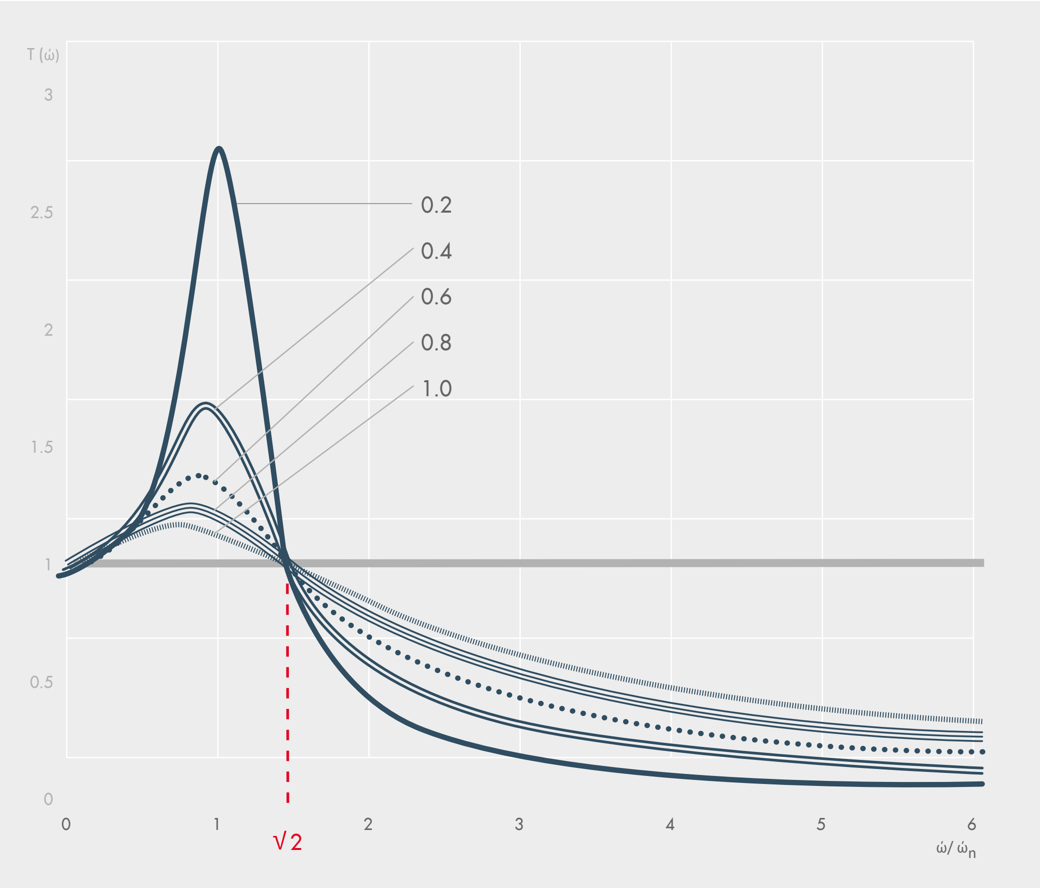
If the oscillating mass m is excited by a periodic force F', the mass supported elastically by vibration isolating elements generates oscillatory motions at an amplitude of $\hat{x}$' based on the following equation:

$$\hat{x} = \frac{F'}{c'\sqrt{\left[1-\left[\frac{f_{err}}{f_0}\right]\right]^2 + 4 \cdot D^2}} \quad [m]$$

D	degree of damping	f_{err}	frequency of the exciter [Hz]
^F'	amplitude of dynamic force [N]	f_0	natural frequency [Hz]
c'	dynamic elastic constant [N/m]	D	degree of damping

The diagram below shows the dynamic increment depending on damping D and the frequency ratio λ between the frequency of the exciter f_{err} and the natural frequency f_0.

TRANSMISSIBILITY CURVE
FOR DIFFERENT DAMPING RATIO



A determining factor for the efficiency of vibration isolation is the ratio between the amplitude of force before (F_err) and after (F_U) the isolation support.

In the case of passive isolation the ratio between the oscillation amplitude x_m of the mass and the amplitude of oscillation x_U of the support base is taken into consideration.

THE ISOLATING EFFECT

The efficiency of an elastic support is above all demonstrated by the damping value K and is calculated on the basis of the following formula.

$$K = 20 \log \left[\sqrt{\frac{1 + 4 \cdot D^2}{\left[1 - \left[\frac{f_{err}}{f_0}\right]^2\right]^2 + 4 \cdot D^2}} \right] \text{[dB]}$$

D degree of damping
f_{err} frequency of the exciter [Hz]
f_0 natural frequency [Hz]

The level of efficiency of the isolation is indicated as a percentage. This percentage refers to an established frequency and is applicable only under certain conditions for the total isolation of an oscillatory system.

$$i = 100 \left[1 - \sqrt{\frac{1 + 4 \cdot D^2}{\left[1 - \left[\frac{f_{err}}{f_0}\right]^2\right]^2 + 4 \cdot D^2}} \right] [\%]$$

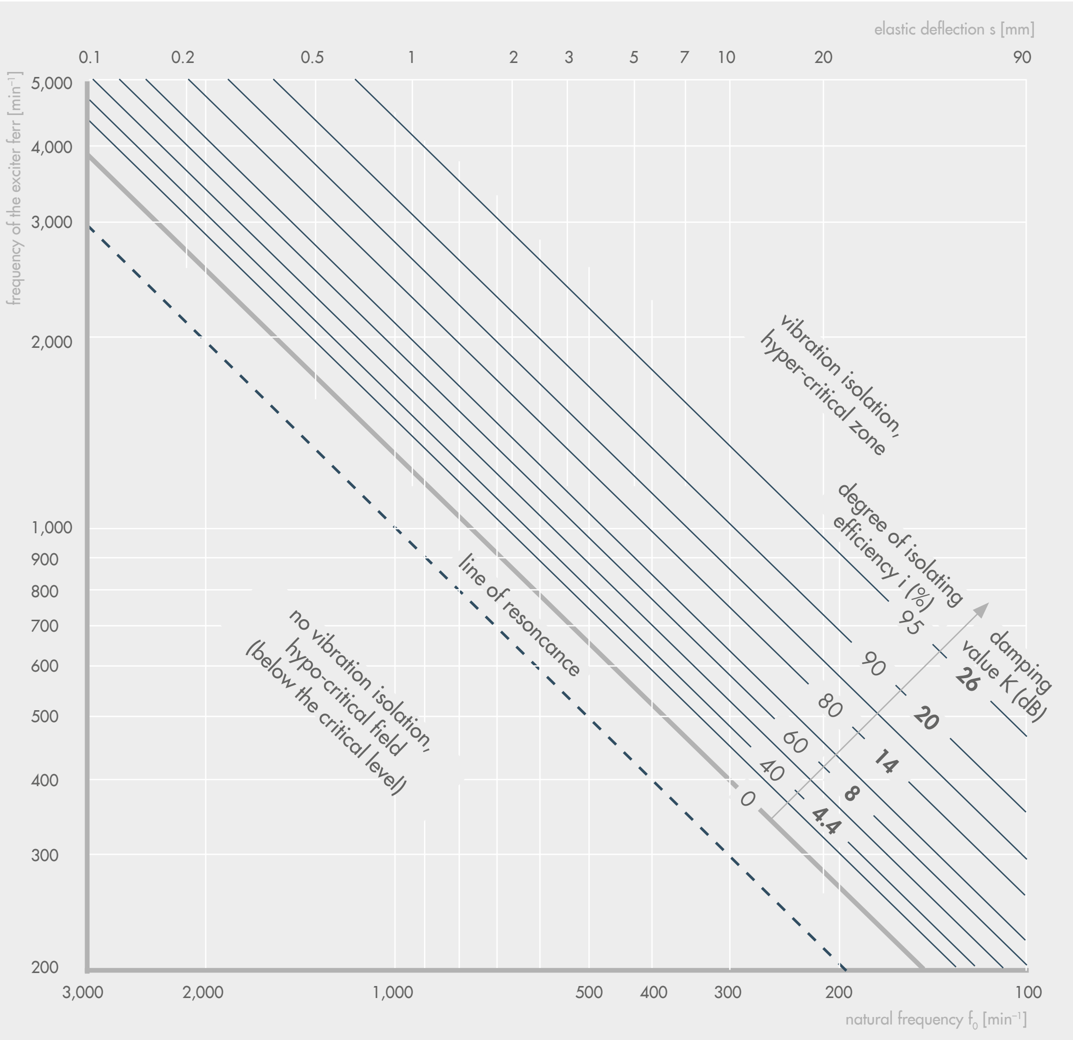
D degree of damping
f_{err} frequency of the exciter [Hz]
f_0 natural frequency [Hz]

THE DEFINITION OF VIBRATION ISOLATING MATERIALS

In order to determine the vibration isolating materials in single objects it is sufficient to establish a uni-dimensional model or make an estimate based on the diagram shown below.

If a more accurate research into oscillatory systems is to be made other possibilities of motion must be allowed. Moreover the oscillating mass can be represented with single masses connected with each other by springs and dampers. Compared with discrete models, a system of this sort has numerous degrees of freedom and natural frequencies formula.

RELATIONSHIP BETWEEN THE FREQUENCY OF THE EXCITER, DAMPING AND ELASTIC DEFLECTION



The effects of vibration

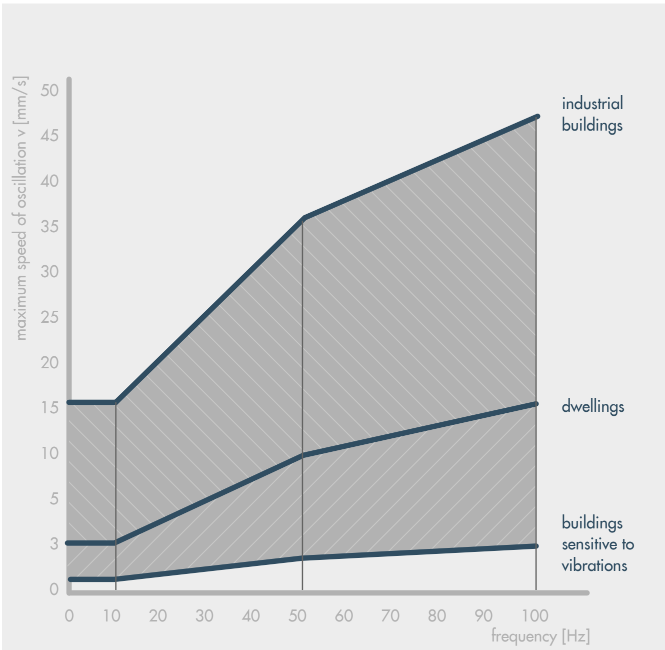
The definition of the approximate values of acceptable vibrations is a complex problem. In the case of machines and plants, the production process must be taken into consideration, while for individuals the definition presents a very wide margin of evaluation. The following physical dimensions are decisive for the evaluation of oscillations:

- Oscillatory amplitude x [mm]
- Speed of oscillation v [mm/s]
- Acceleration of oscillation a [mm/s^2]
- Value KB as a derived dimension

THE EFFECT OF VIBRATIONS ON STRUCTURES

The acceptable speed of oscillation for structures is defined according to Part 3 of the DIN 4150 standard.

Various classes of sensitivity are defined for machines and plants. Detailed indications concerning this can be found in following standards: ISO 2372/73, VDI 2056, VDI 2063, ISA 4.20.



CLASSES OF SENSITIVITY FOR MACHINES AND PLANTS

Classes of sensitivity	Sensitivity with respect for harmonic vibrations	Acceptable amplitudes for frequencies	
		1 ... 10 Hz	10 ... 100 Hz
		Acceleration [mm/s²]	Speed [mm/s]
I	high sensitivity	6.3	0.1
II	average sensitivity	63	1
III	limited sensitivity	250	4
IV	insensitive	> 250	> 4

THE EFFECT OF VIBRATIONS ON PEOPLE

When defining the effect of vibrations on people, the KB value, calculated empirically, is taken into consideration.

The KB value can be calculated by the oscillatory speed v or the acceleration of oscillation a for a specific frequency f.

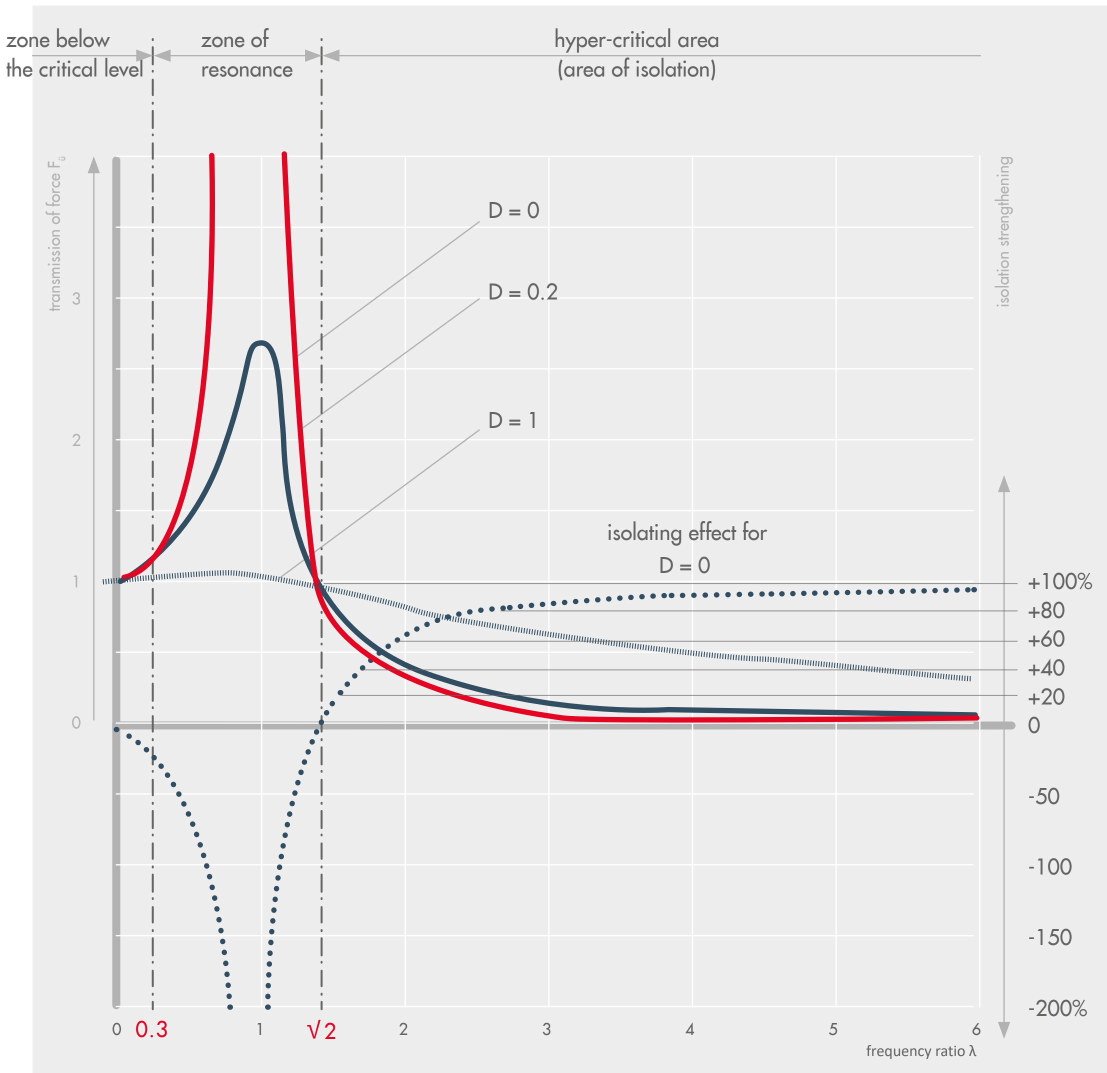
$$KB = x \cdot \frac{0.8 \cdot f^2}{\sqrt{1+0.032 \cdot f^2}} \quad \text{or} \quad x = \frac{v}{2 \cdot \pi \cdot f} = \frac{a}{4 \cdot \pi^2 \cdot f^2} \quad [mm]$$

D degree of damping
f_{err} frequency of the exciter [Hz]
f_0 natural frequency [Hz]

Consequences

RESONANCE CURVE

The ratios between response, damping, isolation, transmission factor and frequency ratio are represented in the following graph, with the so-called resonance curve or amplitude performance.



The ratios of amplitude for active and passive isolation are the same.

The level of efficiency of isolation depends on the frequency and the damping ratios.

Transmission depends above all on the frequency ratio and when over 2 it gives an isolating effect, while it gives a strengthening effect when it is below this value.

General advise how to use antivibration elements

2

Advise for the use of isolating elements

DESIGNING AND PLANNING

Isolating elements are long lasting products. This, however, will only be true if the elements are selected correctly.

In the case of rubber elements it is necessary to consider the fact that, with an effect of equal force, strain will differ according to the type of stress. Most elements can be subjected to pressure, shear and torsion. Short term traction loads deriving from shock effects are acceptable. Continuous traction loads are not acceptable.

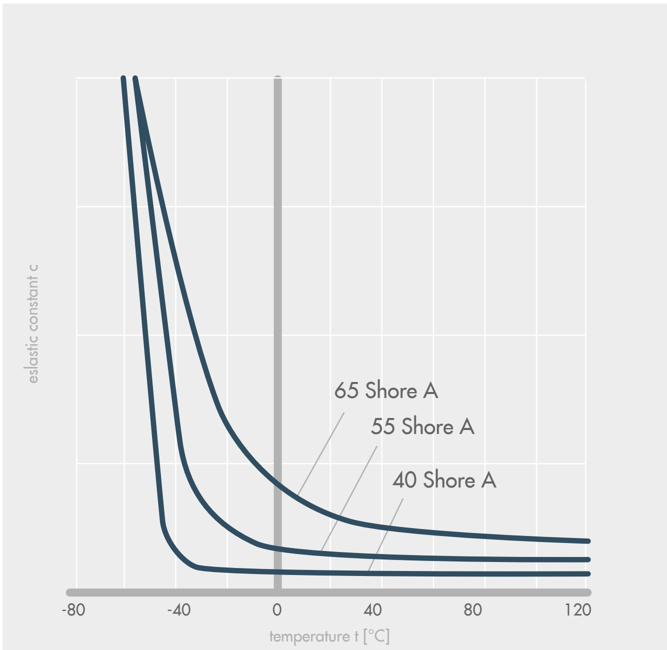
APPROXIMATE LOAD VALUES

Type of load	Acceptable static		Shock
	load (N/mm²)	dynamic (N/mm²)	Air (N/mm²)
pressure	0.5	± 0.125	2.0
shear	0.2	± 0.05	0.6
traction	–	–	1.5
torsion	0.3	± 0.075	0.9
pressure/shear (45°)	0.5	± 0.125	2.0

THE INFLUENCE OF TEMPERATURE

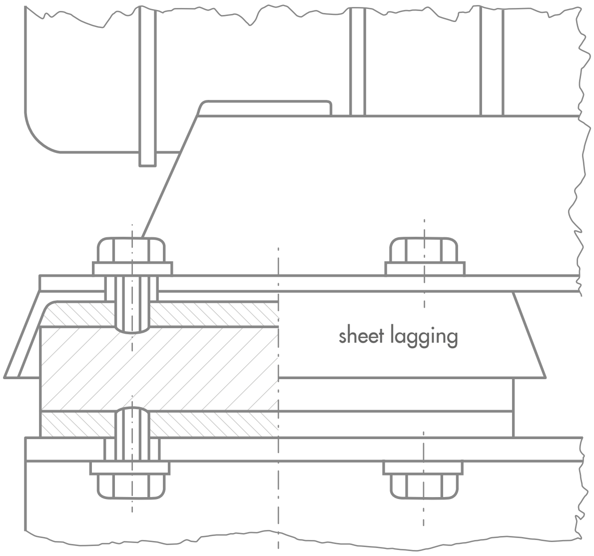
Variations in temperature cause a change in the rigidity of the spring and therefore influence the length of its life.

The following graph shows the elastic constant c as a function of temperature t and shore A hardness.



THE INFLUENCES AND EFFECTS OF CHEMICALS

NR based damping elements are not resistant to the actions of oil, grease, fuel or other chemical products. These elements should be protected by sheathing.



TOLERANCES

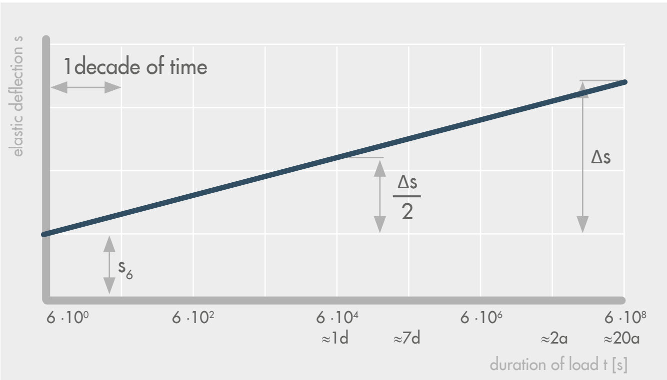
Nominal range of tolerance				Acceptable dimensional deviation							
				Class M 1		Class M 2		Class M 3		Class M 4	
				F	C	F	C	F	C	F	C
				± mm	± mm	± mm	± mm	± mm	± mm	± mm	± mm
mm			6.3	0.10	0.10	0.15	0.20	0.25	0.40	0.50	0.50
to	6.3	to	10.0	0.10	0.15	0.20	0.20	0.30	0.50	0.70	0.70
over	10.0	to	16.0	0.15	0.20	0.20	0.25	0.40	0.60	0.80	0.80
over	16.0	to	25.0	0.20	0.20	0.25	0.35	0.50	0.80	1.00	1.00
over	25.0	to	40.0	0.20	0.25	0.35	0.40	0.60	1.00	1.30	1.30
over	40.0	to	63.0	0.25	0.35	0.40	0.50	0.80	1.30	1.60	1.60
over	63.0	to	100.0	0.35	0.40	0.50	0.70	1.00	1.60	2.00	2.00
over	100.0	to	160.0	0.40	0.50	0.70	0.80	1.30	2.00	2.50	2.50
				%	%	%	%	%	%	%	%
over	160.0			0.30	①	0.50	①	0.80	①	1.50	1.50

① values only on the basis of agreements
F dimension depending on the shape
C dimension not depending on the shape

Acceptable dimensional deviation for elastomer elements:
(taken from standard DIN 7715, part 2)
DIN 7715, part 2, class M 3 can be applied for elastomer elements without special requirements.

PERFORMANCE OF ELASTOMER ELEMENTS WHEN SUBJECTED TO "CREEP"

"Creep" is a characteristic common to elastomer elements. This can be attributed to strain caused by the effect of a load which does not return completely to its original position. In practice, the increase in elastic deflection caused by creep in isolating elements is, in most cases, negligible.



$s6$ elastic deflection after 6 seconds

t duration of load [s] or (decades)

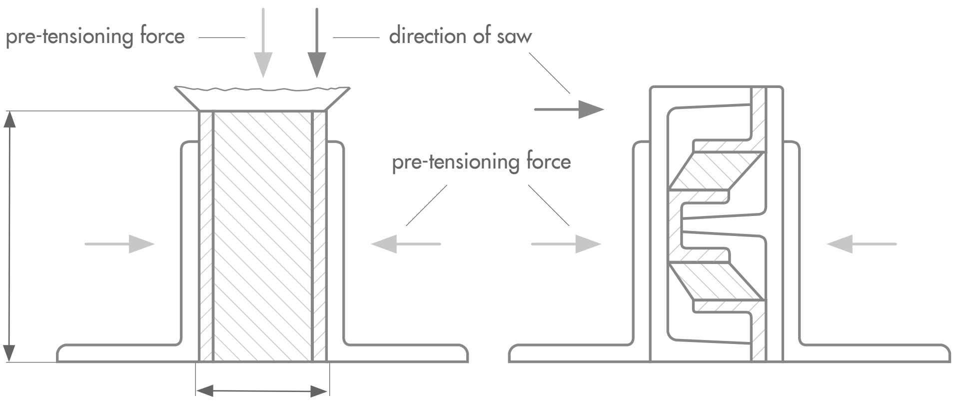
EXAMPLE:

"creep" value of natural rubber 40 Shore A	$K = 0.02$
elastic deflection after 6 seconds	$s_6 = 2mm$
as in graph, we will have: number of decades of time per 1day number of decades of time per 20 years	 $n = 4$ $n = 8$
increase in elastic deflection after 1day	$\Delta s = K \cdot s6 \cdot n$ $\Delta s = 0.02 \cdot 2 \cdot 4$ $\Delta s = 0.16mm$
increase in elastic deflection after 20 years	$\Delta s = K \cdot s6 \cdot n$ $\Delta s = 0.02 \cdot 2 \cdot 8$ $\Delta s = 0.32mm$

MOUNTING ADVICE
PROCESSING RUBBER-METAL ELEMENTS

Rubber-bonded metal bars can be cut to measure using an ordinary commercial band or disc saw. For bars with b < 2h lateral pre-tensioning force can be completed by a higher degree of tightening.

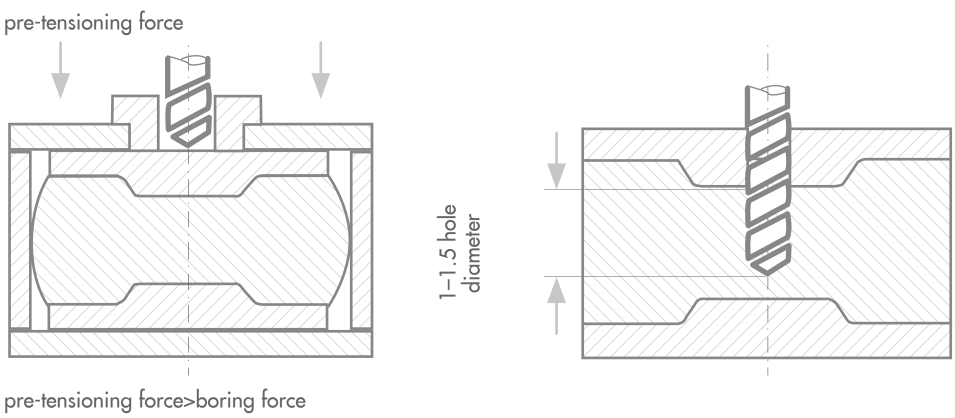
Good lubrication is necessary and good cooling using a coolant-lubricant, mixed with water at the ratio of 1:10. Temperatures above +100°C are not acceptable. Burr must be eliminated after sawing. The joint between the metal and rubber materials along the edges must then be checked but without using sharp objects.



DRILLING AND SCREW THREADING

Drilling and screw threading can usually be carried out the same way as with all metals. The element is attached by its metal part to stop the drill from moving. If it is not possible to attach by the metal part, the rubber-bonded metal element should first of all be tightened with a drilling device or using vices, but always with a pre-tensioning force higher than the drilling force.

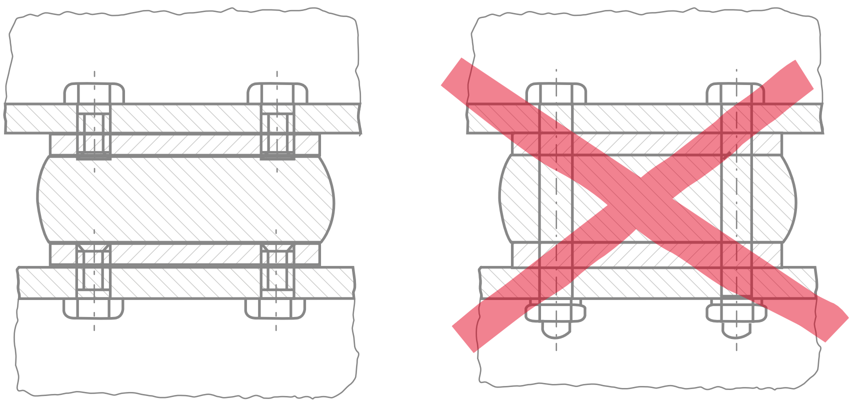
The rubber must not be perforated, and drilling must be done only as shown in the diagram below. You are advised to make deep holes during drilling. Good lubrication and cooling are indispensable, achieved using an appropriate liquid for drills. Temperatures higher than +100°C are unacceptable. The length of the screws must correspond to the thickness of the metal part and must not penetrate into the elastic body.



Rubber-bonded metal elements can be screwed into the ground or to machinery.

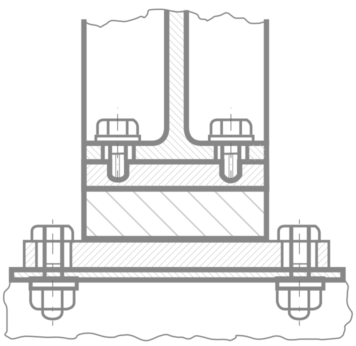
Where the machines are large and exciter forces reduced, it is sufficient to attach the rubber-bonded metal elements to the ground or to the machines themselves. Any unevenness in the surface of the ground can be flattened out using sheets. Under no circumstances should the rubber-bonded metal elements be perforated by screws as this would jeopardise the isolating effect.

If it is necessary to weld the rubber-bonded metal elements using adequate cooling with water, check that both the elastomer and the joint are not exposed to temperatures over +100°C.



ELASTIC CONNECTIONS

The isolating effect of elastic elements must not be compromised by rigid, metal connections. Pipes, shaft couplings and such like must be joined using sufficiently flexible elements. External forces, such as, for example, the traction of a belt, which are not absorbed by support elements, must be intercepted by extra buffers. All elastic connections, including belt traction and buffers used for elastic limitation have an influence when calculating support and must be taken into consideration when calculating vibration. Buffers and rubber-bonded metal bars which are only statically shear-loaded, must be very slightly pre-compressed to compensate the resultant traction component.



Calculating oscillatory systems Examples of calculations

3

Calculations

ISOLATING SYSTEMS

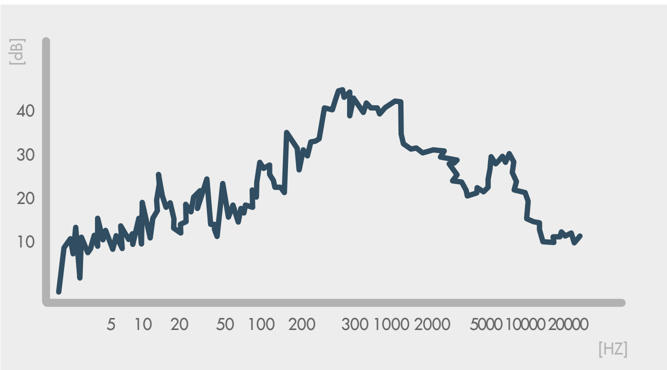
The task of vibration isolation is that of installing an object and connecting it to the environment using isolating elements in such a way as to guarantee that it will function and to prevent any disturbance from being transmitted from or to the environment.

Important conditions for the installation of vibration isolation is a perfect knowledge of the effect of disturbance, the dynamics of the machine, dynamic rigidity and the damping of support elements as well as reciprocal influences.

Isolating elements are defined on the basis of the measurement of oscillations. A reliable definition of optimal isolating elements is only possible by measuring oscillation or force on the three main axes x, y, z followed by an analysis of the data.

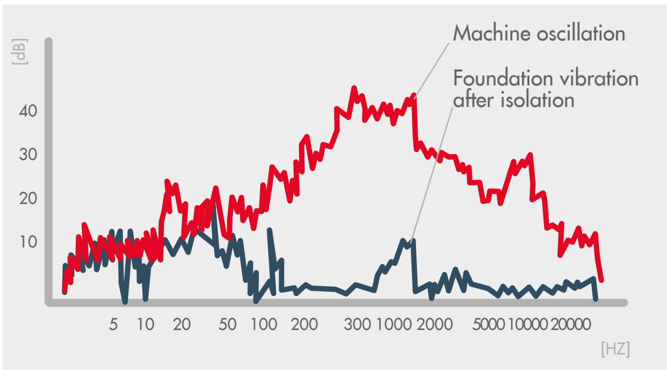
The analysis of frequency indicates the frequency on the axis x and the speed of oscillation on the axis y in decibels (dB). The decibel is used in electronics, acoustics and in oscillation technology.

EXAMPLE: THE FREQUENCY SPECTRUM OF AN ELECTRIC MOTOR



From this analysis of frequencies it is obvious that, in the presence of an isolating support with a natural frequency range from 8 to 12 Hz, all oscillations over 15 Hz would be reduced and it would be possible to achieve a total damping effect of approx. 20 dB.

GRAPHIC REPRESENTATION OF THE ISOLATING EFFECT



The curve a indicates the spectrum of the oscillatory frequency produced by an electric motor while the curve b represents the spectrum of frequency measured on the support base after isolating the motor.

ANALYSIS

It is obvious, from the measurement, that in the support range of natural frequency of 12 Hz there is a slight amplification while in the range starting from 20 Hz isolation is considerable. By defining the average value of the two curves x–M and x–F it is possible to establish the damping value of the isolation on the basis of the following formula:

$$K = 20\log\left[\frac{\bar{x}_M}{\bar{x}_F}\right][\mathrm{dB}]$$

$\bar{x}_M$ average value of the machine's oscillations
$\bar{x}_F$ average value of the oscillations of the fundations

$$\text{natural frequency: } f_0 = f_{err}\sqrt{\frac{100 - i}{200 \quad i}}\,[\mathrm{Hz}]$$

f_{err} exciter frequency [Hz]
i degree of efficiency of isolation [%]

$$f_0 = \frac{1}{2\cdot\pi}\sqrt{\frac{c'}{m}}\,[\mathrm{Hz}]$$

$$c' = \frac{E'\cdot A\cdot 1000}{h}\,[\mathrm{N/mm}]$$

c' dynamic elastic constant [N/mm]
m mass [kg]
E' modulus of dynamic elasticity [N/mm²]
A section [mm²]
h thickness of the material [mm]

ISOLATING ELEMENTS

In practice, it is very difficult to calculate isolating elements because, in most cases, the necessary basic information regarding forces of equilibrium, the position of the centre of gravity, the rigidity of the machine and the acceptable oscillation speeds are not available. Moreover, these influences can vary considerably even in machines of the same structure.

In order to understand better the relationships, the following calculations refer to a single mass oscillator with excitation of harmonic force and an extremely rigid installation position (something which never actually happens). This simplification is, however, acceptable for many plants.

The exciter frequency f_{err} of the supported object is always a decisive factor in establishing the efficiency of an isolating support where vibrations exist. The necessary natural frequency f_0 of isolating elements can be calculated by the degree of efficiency of isolation i indicated or recorded.

When calculating the degree of efficiency of isolation it is necessary to bear in mind that one hundred per cent isolation is not possible. The economic limit falls between 80% and 95%. Better levels of isolation are possible using specific applications for supplementary foundations, pneumatic supports, etc.

FLAT SUPPORTS

Completely flat isolating supports are calculated by means of the modulus of dynamic elasticity E' of the material used.

The modulus of dynamic elasticity E' depends on frequency. In creating flat supports it is necessary to ensure that no other supports or rigid connections exist.

ELASTOMER ELEMENTS

For elastomer elements the curves which are mostly to be found are those of load-deflection (buffers, bars, support elements). From these curves it is possible to record the elastic deflection s_{sub} as a function of the load.

It is necessary to bear in mind in the case of elastomer elements that the indicated elastic curves are based on almost static measurements and that in the presence of higher frequency the elements present greater rigidity. Compression loads can cancel tangential elasticity (fluctuation).

$$f_0 \approx 60 \sqrt{\frac{250}{s_{sub}}} \quad [min^{-1}]$$

$$f_0 \approx \frac{\sqrt{250}}{s_{sub}} [Hz]$$

f_0 natural frequency [Hz]

s_{sub} linearised elastic deflection [mm]

ELEMENTS WITH STEEL SPRINGS

The natural frequency of steel springs can be calculated very accurately because they have a reduced production tolerance and a linear elastic characteristic.

$$f_0 \approx 60 \sqrt{\frac{250}{s}} \quad [min^{-1}]$$

$$f_0 \approx \frac{\sqrt{250}}{s} [Hz]$$

f_0 natural frequency [Hz]

s elastic deflection [mm]

Spiral springs have a very low level of damping (D = 0.004) and are mostl.

The natural frequency of elements with steel springs can be obtained directly from the technical sheets. Intermediary values can therefore be defined by interpolation. Elements with steel springs should also be fitted with damping sheets because they have the tendency to develop a natural oscillatory state.

ELEMENTS WITH PNEUMATIC SPRINGS

As can be demonstrated by applying and converting the basic equation for natural frequency, the natural frequency of pneumatic springs depends exclusively on the respective height h.

Pneumatic springs can be adapted to various load patterns by means of variations in pressure.

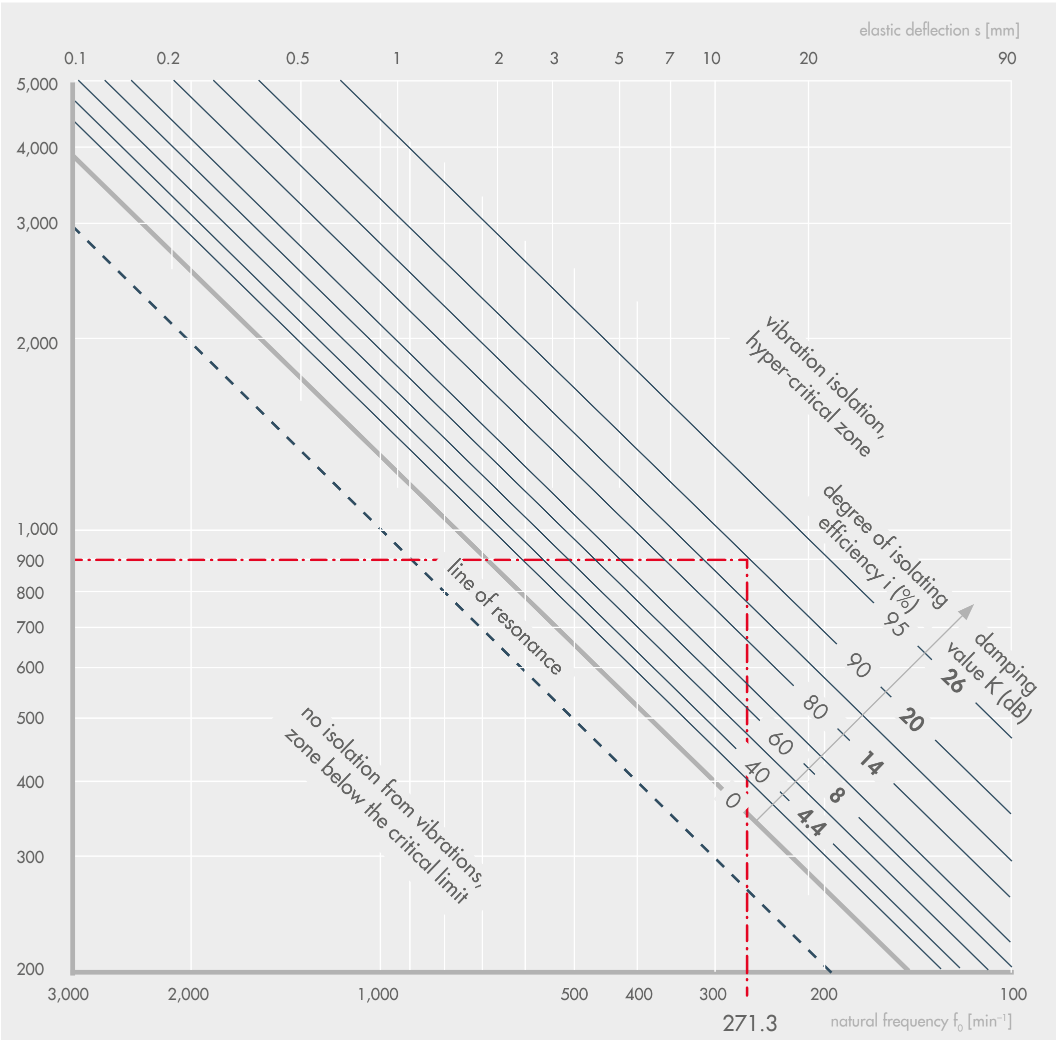
$$f_0 = \frac{1}{2 \cdot \varpi} \sqrt{\frac{g \cdot \chi}{h}} \text{ [Hz]}$$

- **g** acceleration of gravity [9.81m/s2]
- **χ** isentropic index (ratio of the values of specific heat in the air with constant volume and pressure)
- **h** height [m]

The natural frequencies f_0 of the elements are given in the technical sheets as a function of pressure and load.

DEFINITION OF THE NECESSARY ELASTIC DEFLECTION OF AN ISTOLATING ELEMENT

If the disturbance frequencies and the necessary degree of efficiency of isolation are known, it will be possible to define the natural frequency of the elements or the respective linear elastic deflection, as in the graph shown below.



TRANSMISSION OF FORCE TO THE ENVIRONMENT

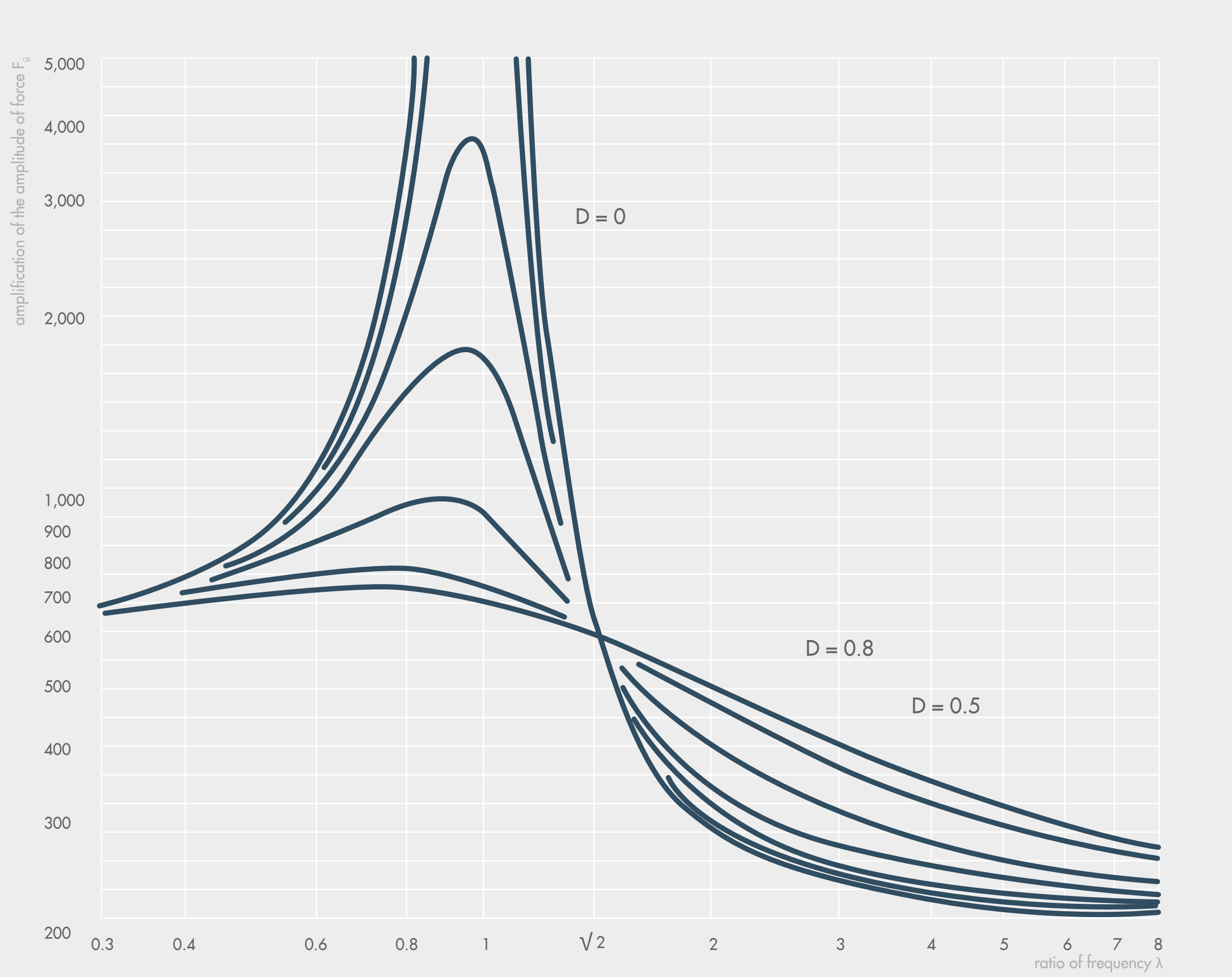


Diagram of the coefficient of transmission of force $F_ü$ as a function of the ratio of frequency λ and damping D.

Formula for calculting the coefficient of transmission of force $F_ü$:

$$F_ü = \sqrt{\frac{1+4 \cdot D^2 \cdot \lambda^2}{(1-\lambda^2)^2 + 4 \cdot D^2 \cdot \lambda^2}}$$

λ ratio of frequency
D degree of damping

Zone: λ = 0 ÷ 0.3
In this zone the plant is connected to the environment in a more or less rigid manner and the forces of disturbance are transmitted.

Zone: λ = 0.3 ÷ √2
In the zone of resonance, depending on the damping of the support elements, it is possible to transfer from twice to ten times the force of disturbance to the environment.

Zone: λ = √2 ÷ 8
In the zone of isolation λ > √2, depending on the damping of the support elements, it is possible to transfer only a small part of the forces of disturbance to the environment.

THE OSCILLATORY MOTION OF A PLANT

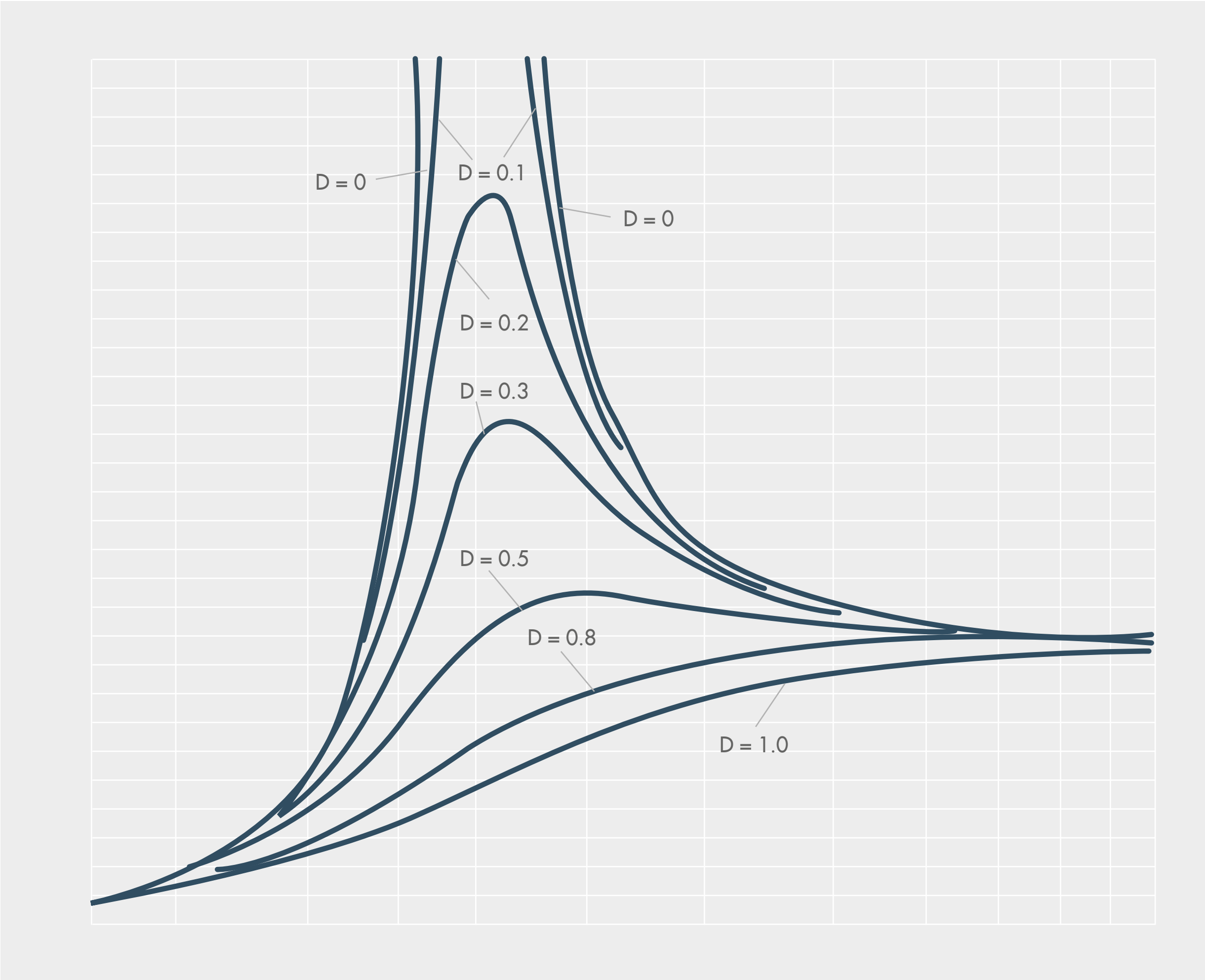


Diagram of the coefficient of displacement amplification (increase of deflection) $x_{ü}$ as a function of the ratio of frequency λ and damping D.

Formula for calculating the coefficient of deflection amplification $x_{ü}$:

$$x_{ü} = \sqrt{\frac{\lambda^2}{(1-\lambda^2)^2 + 4 \cdot D^2 \cdot \lambda^2}}$$

λ ratio of frequency
D degree of damping

Zone: λ = 0.4 ÷4
In this zone the amplitude of oscillation is reduced depending on the damping of the support elements.

Zone: λ = 4÷8
In the zone of isolation the amplitude of oscillation of the plant depends only on the ratio of the force of disturbance with respect to the mass of the machine.

I.e. the damping of the support elements in the zone λ > 4 has practically no effect on the vibrations of the plant.

Examples of calculations

STRAIN IN ELASTOMER PANELS

Elastomer panels and strips are suitable for compensating any possible misalignment. They do however have a limited isolating effect against vibrations. Calculations concerning elastomer supports cannot be based solely on the acceptable specific load as in the case of steel or concrete. The following factors must be taken into consideration:

- the coefficient of shape based on the geometry of the elastomer element
- the hardness of the elastomer panel
- the percentage of strain
- the coefficient of friction between the elastomer and the support point
- the direction of load

CALCULATION FOR COMPRESSION STRESS

Elastic deflection s_D is calculated by the relation:

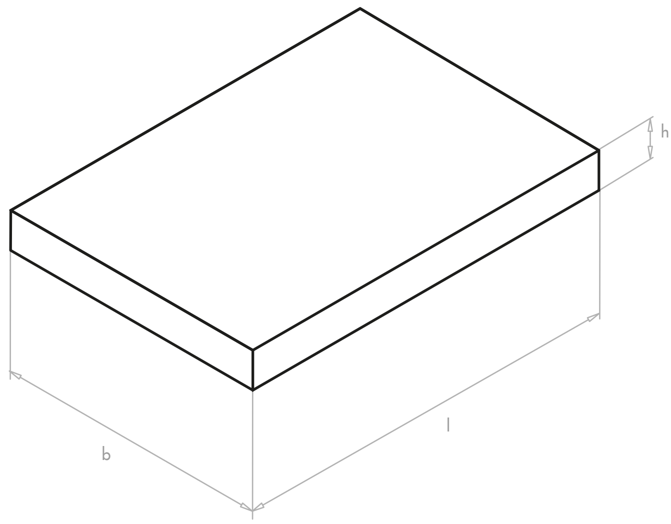
$$s_D = \frac{F \cdot h}{A \cdot E_C} \text{ [mm]}$$

From this it is possible to calculate the elastic compression constant:

$$C_D = \frac{F}{s_D} = \frac{A \cdot E_C}{h} \text{ [N/mm]}$$

F force [N]
h thickness of the panel [mm]
A support surface [mm²]
EC modulus of compression [N/mm²]

As an approximate value here, the elastic deflection s_D should not be higher than 10-15% of the thickness of the panel h. The maximum value will be s_D = 20% of the thickness of the panel h.



THE COEFFICIENT OF SHAPE

The coefficient of shape q must, above all, be calculated from the dimensions. The coefficient of shape q is defined as the ratio of the loaded surface with respect to the surrounding surface without lagging.

$$q = \frac{b \cdot l}{2h\,(b + l)}$$

If l >> b, the above mentioned equation will be reduced by the coefficient of shape q to:

$$q = \frac{b}{2 \cdot h}$$

In the case of hollow cylinders, the coefficient of shape q is calculated on the basis of the following formula:

$$q = \frac{D - d}{4 \cdot h}$$

or, similarly, for solid cylinders:

$$q = \frac{D}{4 \cdot h}$$

D external diameter [mm]
d internal diameter [mm]
h thickness of the sheet or height of the cylinder [mm]

The following calculation can be applied only if this coefficient of shape is lower than 3. Following this the isolating effect of these sheet supports is limited to disturbance frequencies which are higher than 60 Hz (solid borne noise).

THE MODULUS OF COMPRESSION

When calculating the modulus of compression EC the modulus of sliding G must also be calculated. The following ratio exists between the hardness H and the modulus of sliding G:

$$G = 0.086 \cdot 1.045^{H}\ [N/mm^2]$$

H hardness [Shore A]

The modulus of compression EC depends on both the connection between elastomer and support surface and the coefficient of shape q and the modulus of sliding G. Two possibilities are therefore available when calculating the modulus of compression E_C:

– E_C with free contact bearing surface:

$$E_C = 3 \cdot G\,(1 + q + q^2)$$

– E_C with joined, vulcanised contact surfaces:

$$E_C = 3.3 \cdot G\,(1 + q + q^2)$$

CALCULATING THE SHEAR STRESS

If in the equation of elastic deflection s_D for compression strain:

$$s_D = \frac{F \cdot h}{A \cdot E_C} \text{ mm}$$

F	force [N]	A	support surface [mm²]
h	thickness of the plate [mm]	E_C	modulus of compression [N/mm²]

we replace the modulus of compression E_C with the modulus of sliding G:

$$G = 0.086 \cdot 1.045^H \text{ [N/mm}^2\text{]}$$

H hardness [Shore A]

the elastic deflection for shear s_S (displacement) will be obtained:

$$s_s = \frac{F \cdot h}{A \cdot G} \text{ [mm]}$$

F	force [N]	A	support surface [mm²]
h	thickness of the plate [mm]	G	modulus of sliding [N/mm²]

or the elastic shear constant c_S:

$$s_s = \frac{F \cdot h}{A \cdot G} \text{ [mm]}$$

F	force [N]	A	support surface [mm²]
h	thickness of the plate [mm]	G	modulus of sliding [N/mm²]

The approximate value for the acceptable elastic deflection of shear will be s_S = 25% of the thickness of the panel h.

The maximum value will be s_S = 35% of the thickness of the panel h.

EXAMPLE

Calculation of the elastic deflection s_D of a rubber plate with a free support (neoprene 70) with a load F of 1000 kg:

hardness H = 70 Shore A
length l = 200 mm
width b = 100 mm
thickness h = 20 mm
force F = 10000 N

Coefficient of shape q =

$$q = \frac{b \cdot l}{2 \cdot h\,(b + l)} - \frac{100 \cdot 200}{2 \cdot 20\,(100 + 200)} = 1.66$$

Modulus of sliding:

$$G = 0.086 \cdot 1.045^{H} = 0.086 \cdot 1.045^{70} = 1.87\ N/mm^2$$

Depending on the task to be carried out this is an isolating plate with a free support. Modulus of compression:

$$EC = 3 \cdot G(1+q+q^2) = 3 \cdot 1.87(1+1.66+1.66^2) = 30.38 N/mm^2$$

Elastic deflection

$$s_D = \frac{F \cdot h}{A \cdot E_C} = \frac{10{,}000 \cdot 20}{20{,}000 \cdot 30.38} = 0.33mm$$

Monitoring the percentage of elastic deflection:

$$s_D[\%] = \frac{s_D \cdot 100}{h} = \frac{0.33 \cdot 100}{20} = 1.65\%$$

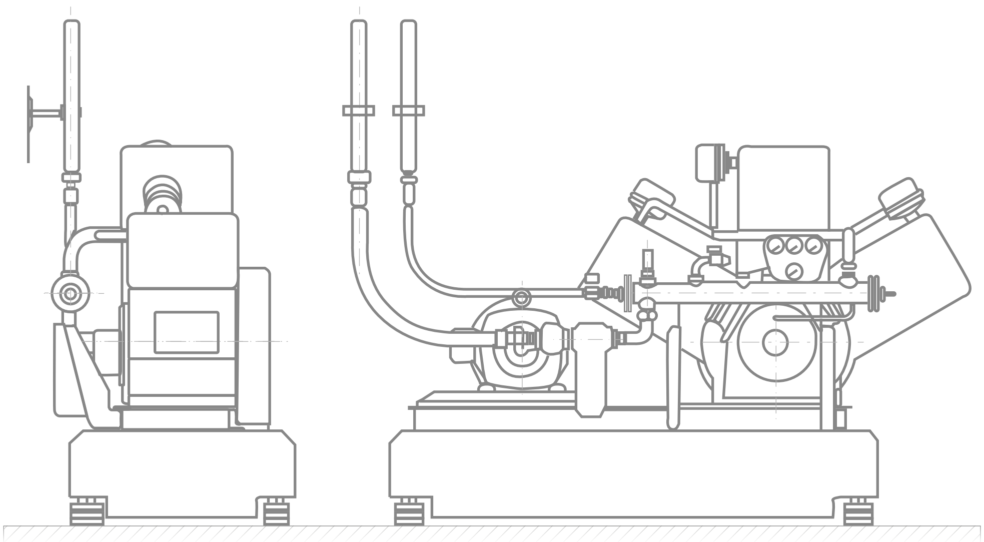
the result falls within the15% value limit.

Calculating the specific load:

$$p = \frac{F}{A} = \frac{F}{b \cdot l} = \frac{10{,}000}{20{,}000} = 0.5N/mm^2$$

SUPPORT ELEMENT

A 3-cylinder compressor must rest on support elements as shown in the diagram below:



Data:

– mass of the electric motor with accessoires	m_1= 20kg
– mass of the compressor	m_2= 400 kg
– number of revs of the compressor	n_2 = 900 min^{-1}
– number of revs of the motor	n_1 = 1450 min^{-1}
– number of damping elements	6
– isolation effect	90%

To be obtained:

- counter-mass requested a foundation base
- centre of gravity of the plant
- layout of the damping elements
- type of element
- measures to be taken to prevent the transmission of vibrations via the cooling piping or forced air piping.

SOLUTION

Machines with great moving masses, such as this compressor, for example, produce vibrations which can be transmitted to the structures of the building or cause annoying noise or breakage if there are no damping supports.

DEFINITION OF EXTRA COUNTER-MASS

The amplitude of oscillation of a supported plant is reduced thanks to extra counter-mass. In the case of slow-rotating compressors, in particular, a foundation base equal to three to five times the weight of the compressor will be required.

Pre-calculated data:
concrete foundation with the following measurements:

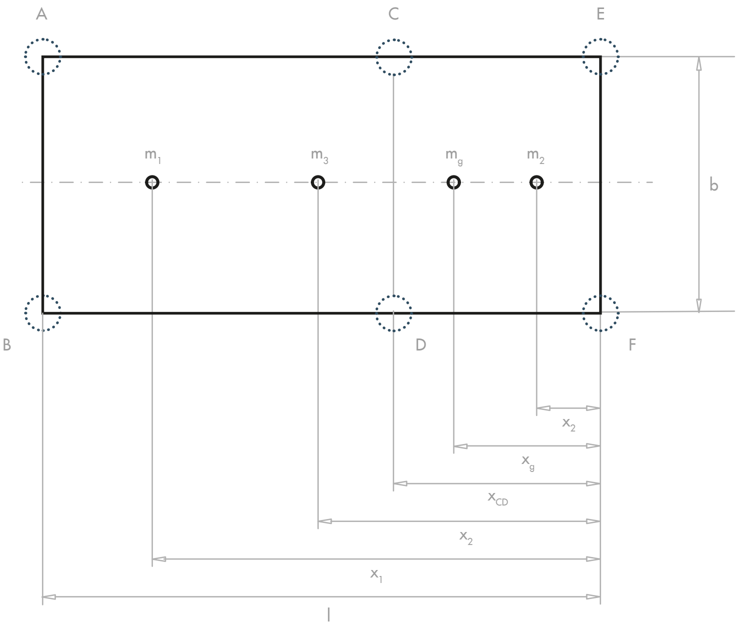
length $l = 2200mm$
width $b = 1100mm$
thickness $h = 300mm$
density of the reinforced concrete
$\rho = 2.3\ kg/dm^3$

the foundation will have a mass of m^3

$$m^3 = l \cdot b \cdot h \cdot \rho$$

$$m^3 = 22 \cdot 11 \cdot 3 \cdot 2.3 = 1669.8kg$$

DEFINITION OF THE MEASUREMENTS



DEFINITION OF THE LAYOUT OF THE SUPPORT ELEMENTS

Calculation of the position of the general centre of gravity:

x_1 = distance from the centre of gravity per m_1 = 1600mm
x_2 = distance from the centre of gravity per m_2 = 200mm
x_3 = distance from the centre of gravity per m_3 = 1100mm

Weight of the plant:
$mg = m_1 + m_2 + m_3$
$mg = 20 + 400 + 1669.8 = 2089.8kg$

Position of the centre of gravity:

$$x_g = \frac{(x_1 \cdot m_1) + (x_2 \cdot m_2) + (x_3 \cdot m_3)}{m_g}$$

$$x_g = \frac{(1{,}600 \cdot 20) + (400 \cdot 200) + (1{,}100 \cdot 1{,}669.8)}{2{,}089.8} = 932.5mm$$

In case of an asymmetric, transversal position of the weights, the co-ordinate y of the centre of gravity must be calculated in a similar way. For the sake of simplicity, in this example a symmetric, transversal layout has been considered.

LAYOUT OF OSCILLATING ELEMENTS

According to its function the plant must be supported by 6 elements. The layout of the elements must permit an even distribution of the load on all the elements.

$$m_E = \frac{m_g}{6} = \frac{2089.8\ \text{kg}}{6} = 348.3\ \text{kg}$$

Load for each element, in the presence of 6 elements:

$$\Sigma M_{FE} = 0 = 2 \cdot m_E \cdot l - m_g \cdot x_g + 2 \cdot m_E \cdot x_{CD}$$

$$x_{CD} = \frac{-2 \cdot m_E \cdot l + m_g \cdot x_g}{2 \cdot m_E} = \frac{-2 \cdot 348.3 \cdot 2{,}200 + 2{,}089.8 \cdot 932.5}{2 \cdot 348{,}3} = 597.5\ \text{mm}$$

CHOICE OF SUPPORT ELEMENTS WITH A 90% DEGREE OF ISOLATION EFFICIENCY

Calculation of the necessary natural frequency value f_0.

The exciter frequency f_{err}, which must be taken into consideration when defining the natural frequency f_0, is obtained from the speed of the compressor n_2.

The formula for the degree of isolating efficiency i:

$$i = \frac{\left[\frac{f_{err}}{f_0}\right]^2 - 2}{\left[\frac{f_{err}}{f_0}\right]^2 - 1} \cdot 100 = \frac{\lambda^2 - 2}{\lambda^2 - 1} \cdot 100\ [\%]$$

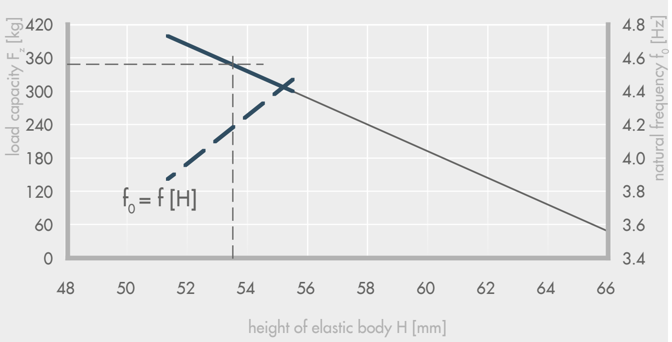
broken down on the basis of f_0 we have:

$$f_0 = f_{err} \sqrt{\frac{100-i}{200-i}}$$

$$f_0 = 900 \sqrt{\frac{100-90}{200-90}} = 271.36\ \text{min}^{-1}$$

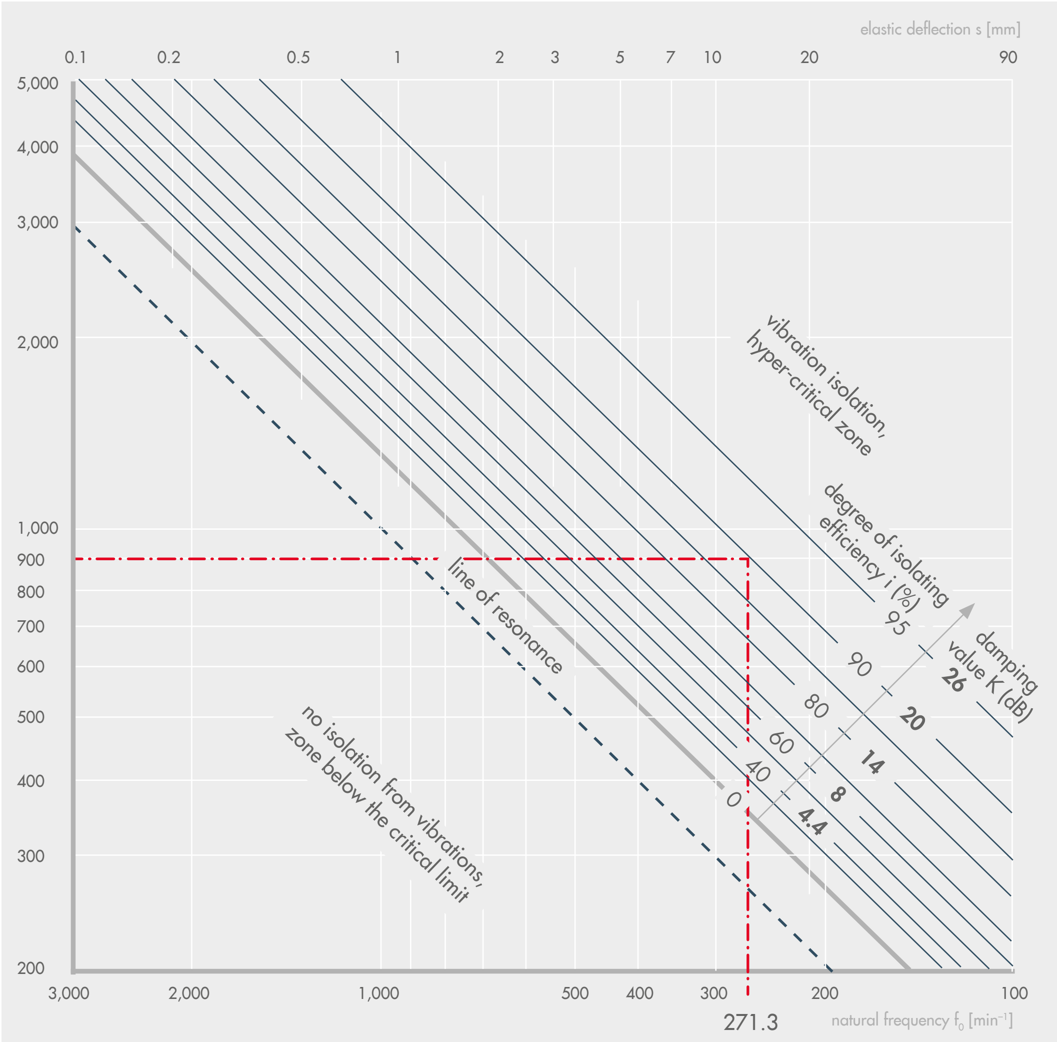
We must then look for support elements which have a natural frequency of 271.3 min–1 or 4.5 Hz, in the presence of a load of 348.3 kg.

The following element was chosen:
GERB® spring element,
model S3Q-244S,
Art. no. 12.2155.0324



It is obvious, from the previous graph concerning the chosen elastic element, that there is a natural frequency of 4.2 Hz with a load capacity F_z of 348.3 kg.

MONITORING THE DEGREE OF ISOLATION EFFICIENCY BY MEANS OF A GRAPH



MEASURES FOR PREVENTING THE TRANSMISSION OF OSCILLATIONS BY SECONDARY WAYS

- Obviously the natural frequency of the place of installation (ground) must be higher than the frequency of the exciter of 15 Hz.
- There must be no fixed connections between the plant and the environment.
- Flexible connections must be sufficiently long and must be mounted with a 90° elbow.

ISOLATING ELEMENTS

Electrical control apparatus must be mounted onto a transport system. Isolation must be provided in order to prevent problems of electronic nature.

Known data:

- weight of the control apparatus m = 60 kg
- number of fix points: 4
- exciter frequency of the transport system f_{err} = 1450 min–1
- desired isolating effect, very good (> 80%)

To be found:
an isolating element to be fixed to the wall

DEFINITION OF THE ELASTIC DEFLECTION NECESSARY FOR THE SUPPORT ELEMENTS

The approximate equation for natural frequency:

$$f_0 = 60\sqrt{\frac{250}{s_s}}$$

broken down on the basis of elastic deflection, this will give:

$$s_s = \frac{250 \cdot 3{,}600}{f_0{}^2} = \frac{250 \cdot 3{,}600}{591.9^2} = 2.57\text{mm}$$

SOLUTION

The solution required in this case is of the passive isolation type, which must be capable of protecting the electronic apparatus from potential external disturbance.

Use of a wall fixture means that the elements to be isolated must be chosen according to shear.

DEFINITION OF THE LOAD EACH OF FIXING ELEMENT

$$F = m \cdot g = 60 \cdot 9.81 \approx 600\text{N}$$

$$F_{1\text{-}4} = \frac{F}{4} = \frac{600}{4} = 150\text{ N}$$

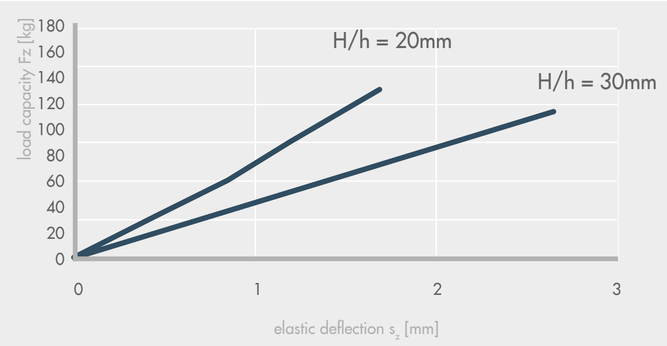
The formula used for the degree of isolating efficiency, broken down on the basis of f_0, will give the value of natural oscillation required for a degree of isolation of 80%:

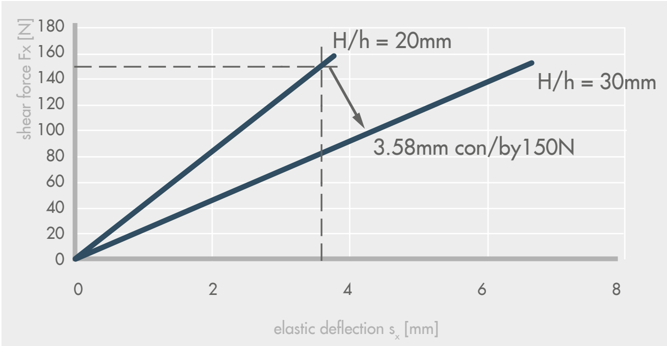
$$f_0 = f_{err}\sqrt{\frac{100\text{-}i}{200\text{-}i}} = 1{,}450\sqrt{\frac{100\text{-}80}{200\text{-}80}} = 591.9\text{ mm}^{-1}$$

The following has been chosen:

- cylindrical buffer model A
- Art. no. 12.2001.6903
- hardness: 57 Shore A
- dimensions: Ø 30 x 20mm
- threaded pins: M8 x 20mm

According to their elastic shear characteristic, with a load of 150 N, the selected cylindrical buffers will produce an elastic deflection s_S of 3.58 mm.





We will therefore have a natural frequency of:

$$0 = 60\sqrt{\frac{250}{s_s}} = 60\sqrt{\frac{250}{3.58}} = 501.51\,\text{min}^{-1}$$

Verification of the isolating effect:

$$i = 100\left(1 - \frac{1}{\left(\frac{f_{err}}{f_0}\right)^2 - 1}\right) = 100\left(1 - \frac{1}{\left(\frac{1,450}{501,51}\right)^2 - 1}\right) = 86.41\%$$

The degree of isolating efficiency i is equal to 86.41% and therefore higher than the 80% required.

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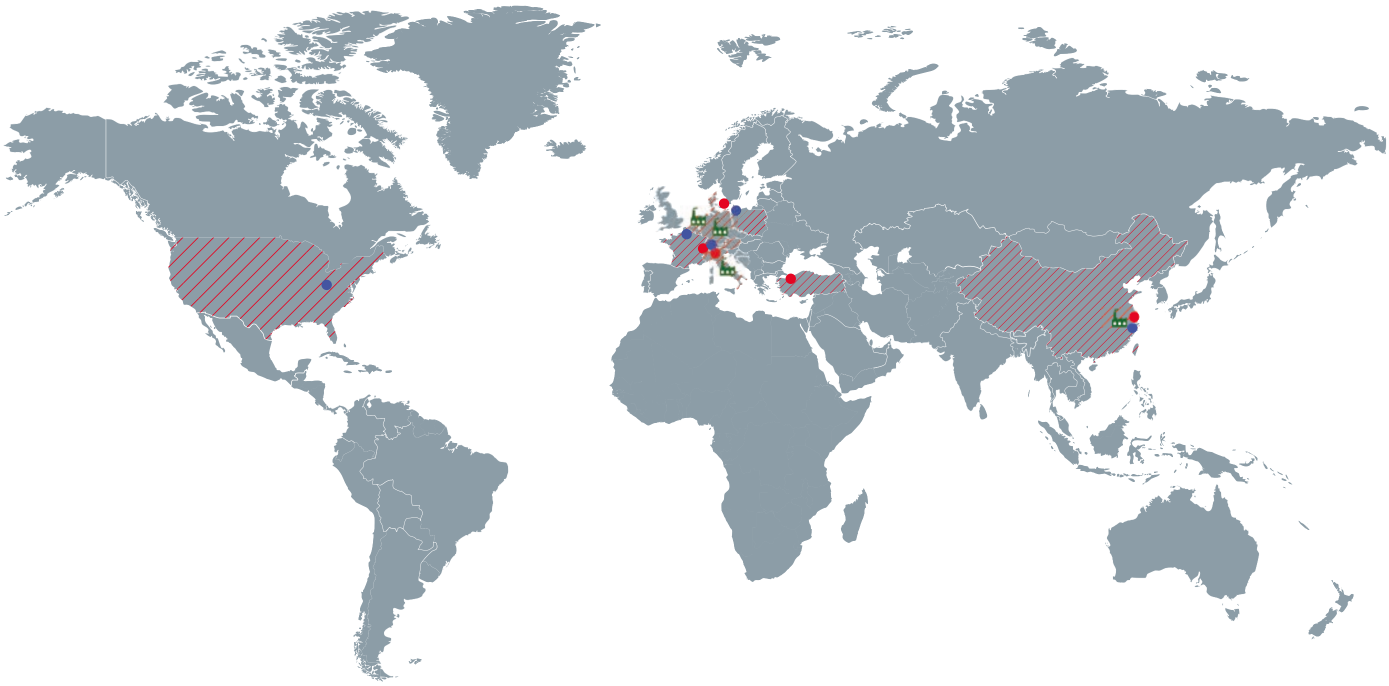

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